Assignment 1

1. For a neuron with a surface area of 0.025 mm^2, a specific membrane capacitance of $c_\mathrm{m} = 10$ nF/mm^2, a specific membrane resistance of $r_\mathrm{m} = 1$ MΩ·mm^2, and a resting membrane potential $E = -70$ mV:

   a) What is the total membrane capacitance C_m?

   b) What is the total membrane resistance R_m?

   c) What is the membrane time constant τ_m?

   d) How much external electrode current would be required to hold the neuron at a membrane potential of -65 mV?

   e) If this amount of current is turned on at time $t = 0$, with the cell initially at -70 mV, and held constant at this value, at what time t will the neuron reach a membrane potential of -67 mV?

2. Build an integrate-and-fire model neuron,

$$\tau_\mathrm{m} \frac{dV}{dt} = V_\mathrm{rest} - V + R_\mathrm{m} I\,.$$

   With $V_\mathrm{rest} = V_\mathrm{reset} = -65$ mV, $V_\mathrm{th} = -50$ mV, $\tau_\mathrm{m} = 10$ ms, and $R_\mathrm{m} = 10$ MΩ. Reset the potential to $V = V_\mathrm{reset}$ whenever it goes to or above V_th and the neuron fires an action potential. Apply different levels of constant current I and compare your results to the analytic formula for the rate of action potential generation:

$$r = \left(\tau_\mathrm{m} \ln\left(\frac{R_\mathrm{m} I_\mathrm{e} + V_\mathrm{rest} - V_\mathrm{reset}}{R_\mathrm{m} I_\mathrm{e} + V_\mathrm{rest} - V_\mathrm{th}}\right)\right)^{-1}\,.$$

3. Construct an integrate-and-fire model with an excitatory synaptic conductance based on the equation,

$$c_\mathrm{m} \frac{dV}{dt} = -\overline{g}_\mathrm{L}(V - E_\mathrm{L}) - \overline{g}_\mathrm{ex} s(V - E_\mathrm{ex})$$

   with $c_\mathrm{m} = 10$ nF/mm^2, $\overline{g}_\mathrm{L} = 1.0\ \mu$S/mm^2, $E_\mathrm{L} = -70$ mV, $\overline{g}_\mathrm{ex} = 0.5\ \mu$S/mm^2 and $E_\mathrm{ex} = 0$. Also, the threshold and reset potentials for the model are $V_\mathrm{th} = -54$ mV and $V_\mathrm{reset} = -80$ mV. The excitatory conductance should satisfy the equation

$$\tau_\mathrm{ex} \frac{ds}{dt} = -s$$

   with $\tau_\mathrm{ex} = 10$ ms. In addition, every time there is a presynaptic action potential,

$$s \rightarrow s + 1\,.$$

Plot V(t) in one graph and the synaptic current, defined as,

$$I_{\text{ex}} = \overline{g}_{\text{ex}} s(V - E_{\text{ex}}),$$

in another. Trigger presynaptic action potentials at times 100, 200, 230, 300, 320, 400, and 410 ms. Explain what you see.

4. Construct an integrate-and-fire model responding to a "noisy" input representing the *in vivo* environment. This model is based on the equation

$$\tau_{\text{m}} \frac{dV}{dt} = V_{\text{rest}} - V(t) + \tau_{\text{m}} \sqrt{2D}\eta(t) + R_{\text{m}} I$$

with $\tau_{\text{m}} = 10$ ms, $V_{\text{rest}} = -56$ mV and $R_{\text{m}} = 10$ MΩ. The threshold and reset potentials for the model are $V_{\text{th}} = -54$ mV and $V_{\text{reset}} = -80$ mV. In your simulation, integrate the above equation using the Euler method with a suitably small Δt. At every time step, draw the value of $\eta(t)$ from a Gaussian (normal) distribution with mean 0 and variance $1/\Delta t$. Choose $D = \sigma_V^2/\tau_{\text{m}}$, where σ_V, like I, is a parameter you will vary as discussed below.

a) Set $I = 0$ and turn off the spike generation mechanism in your model (by setting V_{th} to an extremely large value, for example). Plot the standard deviation of the membrane potential fluctuations that arise from different σ_V values in the range $0 \leq \sigma_V \leq 10$ mV.

b) Turn the spikes back on and plot the average firing rate of the neuron (defined by counting spikes over a sufficiently long time interval and dividing by the duration of that interval) as a function of I for $\sigma_V = 2$, 6 and 10 mV. You may have to include negative I values in the range you consider to stop the neuron from firing. How does this differ from the firing-rate curve you determined in problem 2 above?