Assignment 2

1. Build a Hodgkin-Huxley model neuron by numerically integrating the equations for V, m, h, and n:

$$c_\mathrm{m}\frac{dV}{dt} = -i_\mathrm{m} + \frac{I_\mathrm{e}}{A}\,,$$

where

$$i_\mathrm{m} = \overline{g}_\mathrm{L}(V - E_\mathrm{L}) + \overline{g}_\mathrm{K} n^4 (V - E_\mathrm{K}) + \overline{g}_\mathrm{Na} m^3 h (V - E_\mathrm{Na})\,.$$

and

$$\tau_n(V)\frac{dn}{dt} = n_\infty(V) - n\,,$$

$$\tau_n(V) = \frac{1}{\alpha_n(V) + \beta_n(V)}$$

and

$$n_\infty(V) = \frac{\alpha_n(V)}{\alpha_n(V) + \beta_n(V)}\,,$$

and similar equations for m and h, with

$$\alpha_n = \frac{.01(V+55)}{1-\exp(-.1(V+55))} \qquad \beta_n = 0.125\exp(-0.0125(V+65))\,,$$

$$\alpha_m = \frac{.1(V+40)}{1-\exp(-.1(V+40))} \qquad \beta_m = 4\exp(-.0556(V+65))$$

$$\alpha_h = .07\exp(-.05(V+65)) \qquad \beta_h = 1/(1+\exp(-.1(V+35)))\,,$$

In these equations, time is in ms and voltage is in mV. Take c_m = 10 nF/mm^2 and, as initial values, take: $V = -65$ mV, $m = 0.0529$, $h = 0.5961$, and $n = 0.3177$. The maximal conductances and reversal potentials used in the model are $\overline{g}_\mathrm{L} = 0.003$ mS/mm^2, $\overline{g}_\mathrm{K} = 0.36$ mS/mm^2, $\overline{g}_\mathrm{Na} = 1.2$ mS/mm^2, E_L = -54.387 mV, E_K = -77 mV and E_Na = 50 mV. The best method to use to integrate all of these equations over time is the exponential integration scheme discussed in class, but any routine that works is OK. Make your integration step size small enough to get accurate results.

a) Use an external current with $I_\mathrm{e}/A = 200$ nA/mm^2 and plot V, m, h, and n as functions of time for a suitable interval.

b) Plot the firing rate of the model as a function of I_e/A over the range from 0 to 500 nA/mm^2.

c) Apply a pulse of negative current with $I_\mathrm{e}/A = -50$ nA/mm^2 for 5 ms followed by $I_\mathrm{e}/A = 0$ and show what happens. Why does this occur?

2. Extend the Hodgkin-Huxley model you built in the previous problem to a multi-compartment model of an axon. Built the axon out of 100 compartments, each of which has a radius of 2 μm and a length of 100 μm. Each compartment should contain the Hodgkin-Huxley conductances used above. Inject a current into the first compartment of the cable sufficient to evoke action potentials, or use a pulse to evoke a single action potential. All the other compartments should receive no external current. Use the Euler integration method or a Runge-Kutte integration routine (not the exponential scheme used previously) with a time step no larger than around 0.01 ms.

   a) What is the propagation speed of the resulting action potential.

   b) Change the cable radius to 20 μm. What is the propagation speed now? Show that the velocities you get in a and b are proportional to the square-root of the axon radius.