Assignment 8

1. Simulate the neural circuits involved in the random-dot discrimination experiment. The two excitatory populations correspond to the two possible decisions of the monkey (for example left or right). Use the simplified differential equations used in class:

\[
\tau \frac{dr_1}{dt} = -r_1 + F((w_E - w_I)r_1 - w_Ir_2 + h_1)
\]

\[
\tau \frac{dr_2}{dt} = -r_2 + F((w_E - w_I)r_2 - w_Ir_1 + h_2)
\]

\(r_1\) and \(r_2\) are the mean firing rates of the two decision populations. \(F\) is a threshold linear function, \(h_1\) and \(h_2\) are the external inputs (the random dots). \(w_E = w_{EE}\) and \(w_I = w_{EI}\) are the average synaptic weights.

In the simulations start from the initial condition \(r_1 = r_2 = 0\) and tune the parameters so that there are three fixed points, one unstable and two stable, that correspond to the two decisions. Assume that \(h_1\) and \(h_2\) are gaussian variables. Determine the parameters for which the behavior is stochastic when the inputs are perfectly balanced. Determine the psychometric curve by changing the inputs (e.g. by increasing \(h_1\) and decreasing \(h_2\)). Determine the range of the imbalance between \(h_1\) and \(h_2\) in which the behavior is stochastic. This range is usually rather narrow, determine its width.

2. A Hopfield associative memory network has activities for individual units, \(s_i\) for \(i = 1, 2, \ldots, N\), that take values of either +1 or −1, and are updated at every discrete time step of the network dynamics (assume that \(\Delta t = 1\)) by the rule

\[
s_i(t + 1) = \text{sgn} \left( \sum_{j=1}^{N} W_{ij}s_j(t) \right),
\]

where

\[
\text{sgn}(z) = \begin{cases} 
+1 & \text{if } z \geq 0 \\
-1 & \text{if } z < 0 
\end{cases}
\]

\(W_{ij}\) is constructed from \(P\) \(N\)-dimensional “memory” vectors with components \(m_i^a\) (with \(a = 1, 2, \ldots, P\) and \(i = 1, 2, \ldots, N\)), through the outer product

\[
W_{ij} = \sum_{a=1}^{P} m_i^a m_j^a \quad \text{if } i \neq j \quad \text{and} \quad W_{ii} = 0.
\]

Consider a 100-element network \((N = 100)\) and construct \(P\) (specific values of \(P\) will be given below) memory states by randomly assigning +1 and −1 values with equal probabilities to each of the \(N\) elements \(m_i^a\) for each of the \(a = 1, 2, \ldots, P\) memory
vectors. Using these memory vectors, set the matrix of synaptic weights according to equation 2. Then, study the behavior of the network by iterating equation 1.

To measure how close the state of the network at time $t$, given by the variables $s_i(t)$, is to a particular memory state, say $a_1$, define the overlap function

$$q(t) = \frac{1}{N} \sum_{i=1}^{N} s_i(t)m_i^1.$$  

This is equal to 1 if the $s_i = m_i^1$ for all $i$ and is near zero if there is no similarity between the state of the network and the memory vector.

To set the initial values $s_i(0)$ so that a particular value of $q(0)$ is obtained (at least approximately), use the following procedure. For each $i$, set $s_i(0) = m_i^1$ with probability $q(0)$ and otherwise set $s_i(0)$ randomly to either +1 or −1. Plot $q(t)$ as the network evolves from this state according to equation 1. Final values of $q(t)$ near one indicate successful recovery of the memory. Provide sample plots of $q(t)$ from which you obtain the answers to the following questions:

a) What is the range of positive $q(0)$ values that assures successful memory recovery for $P = 1, 5, \text{ and } 10$? Also, determine how accurately the memory is recovered in these three cases by reporting the final value of $q$ obtained from your simulations.

b) Increase $P$ until memory recovery fails ($q$ ends up considerably less than 1) even for $q(0) = 1$. At what $P$ value does this occur?