Assignment 1

1. Classify each of the following differential equations as either linear or nonlinear, and as either autonomous or non-autonomous:
   a) \( \frac{dx}{dt} = a(b - x) + c(t) \)
   b) \( \frac{dy}{dt} = y(1 - y) \)
   c) \( \tau \frac{dV}{dt} = g_1(E_1 - V) + g_2(E_2 - V) \)

2. Consider the following differential equation:

   \[
   \frac{dx}{dt} = -3t^2 x, \quad x(0) = 1.
   \]

   This equation is linear in \( x \), but the coefficient depends on \( t \). Use separation of variables to solve the equation. Observe that the decay is more rapid than for the constant coefficient case.

3. Consider the following differential equation and initial condition:

   \[
   \frac{dx}{dt} = -\frac{1}{2} x + 1 + \sin 3t, \quad x(0) = 5.
   \]

   a) Integrate the equation numerically from \( t = 0 \) to \( t = 5 \) with a step size of \( \Delta t = 0.5 \) and \( \Delta t = 0.01 \) using Euler integration. Plot the two curves on a single graph.

   b) Now do the integration with the exponential integration scheme we discussed in class and \( \Delta t = 0.5 \). Plot the result on the same graph as in (a). Observe that the solution is more accurate than Euler integration with the same step size.

4. For a neuron with a surface area of 0.025 mm\(^2\), a specific membrane capacitance of \( c_m = 10 \) nF/mm\(^2\), a specific membrane resistance of \( r_m = 1 \) M\(\Omega\)-mm\(^2\), and a resting membrane potential \( E = -70 \) mV:
   a) What is the total membrane capacitance \( C_m \)?
   b) What is the total membrane resistance \( R_m \)?
   c) What is the membrane time constant \( \tau_m \)?
   d) How much external electrode current would be required to hold the neuron at a membrane potential of -65 mV?
   e) If this amount of current is turned on at time \( t = 0 \), with the cell initially at -70 mV, and held constant at this value, at what time \( t \) will the neuron reach a membrane potential of -67 mV?
5. Build an integrate-and-fire model neuron,

\[
\tau_m \frac{dV}{dt} = V_{\text{rest}} - V + R_m I.
\]

With \( V_{\text{rest}} = V_{\text{reset}} = -65 \text{ mV}, V_{\text{th}} = -50 \text{ mV}, \tau_m = 10 \text{ ms}, \text{ and } R_m = 10 \text{ M}\Omega \). Reset the potential to \( V = V_{\text{reset}} \) whenever it goes to or above \( V_{\text{th}} \), and then the neuron fires an action potential. Apply different levels of constant current \( I \) and illustrate that your model is working properly.

6. **Optional Problem:** Apply different levels of current \( I \) to your integrate-and-fire model and count spikes over a suitable period of time to compute firing rates. Plot these rate as a function of \( R_m I_e \) and compare your results to the analytic formula

\[
r = \left( \tau_m \ln \left( \frac{R_m I_e + V_{\text{rest}} - V_{\text{reset}}}{R_m I_e + V_{\text{rest}} - V_{\text{th}}} \right) \right)^{-1}.
\]