Assignment 1

1. Consider the following differential equation:

$$\frac{dx}{dt} = -3t^2 x, \;\; x(0) = 1 .$$

   This equation is linear in x, but the coefficient depends on t. Use separation of variables to solve the equation. Observe that the decay is more rapid than for the constant coefficient case.

2. For a neuron with a surface area of 0.025 mm^2, a specific membrane capacitance of $c_\text{m} = 10$ nF/mm^2, a specific membrane resistance of $r_\text{m} = 1$ MΩ·mm^2, and a resting membrane potential $E = -70$ mV:

   a) What is the total membrane capacitance C_m?

   b) What is the total membrane resistance R_m?

   c) What is the membrane time constant τ_m?

   d) How much external electrode current would be required to hold the neuron at a membrane potential of -65 mV?

   e) If this amount of current is turned on at time $t = 0$, with the cell initially at -70 mV, and held constant at this value, at what time t will the neuron reach a membrane potential of -67 mV?

3. Build an integrate-and-fire model neuron,

$$\tau_\text{m} \frac{dV}{dt} = V_\text{rest} - V + R_\text{m} I_\text{e} .$$

   With $V_\text{rest} = V_\text{reset} = -65$ mV, $V_\text{th} = -50$ mV, $\tau_\text{m} = 10$ ms, and $R_\text{m} = 10$ MΩ. Reset the potential to $V = V_\text{reset}$ whenever it goes to or above V_th and the neuron fires an action potential. Apply different levels of constant current I and compare your results to the analytic formula for the rate of action potential generation:

$$r = \left(\tau_\text{m} \ln \left(\frac{R_\text{m} I_\text{e} + V_\text{rest} - V_\text{reset}}{R_\text{m} I_\text{e} + V_\text{rest} - V_\text{th}} \right) \right)^{-1} .$$