Assignment 2

1. Consider the following differential equation and initial condition:

$$\frac{dx}{dt} = -\frac{1}{2}x + 1 + \sin 3t, \;\; x(0) = 5 \, .$$

   a) Integrate the equation numerically from $t = 0$ to $t = 5$ with a step size of $\Delta t = 0.5$ and $\Delta t = 0.01$ using Euler integration. Plot the two curves on a single graph.

   b) Now do the integration with the exponential integration scheme we discussed in class and $\Delta t = 0.5$. Plot the result on the same graph as in (a). Observe that the solution is more accurate than Euler integration with the same step size.

2. Construct an integrate-and-fire model with an excitatory synaptic conductance based on the equation,

$$c_{\rm m}\frac{dV}{dt} = -\overline{g}_{\rm L}(V - E_{\rm L}) - \overline{g}_{\rm ex}s(V - E_{\rm ex})$$

   with $c_{\rm m} = 10$ nF/mm^2, $\overline{g}_{\rm L} = 1.0$ μS/mm^2, $E_{\rm L} = -70$ mV, $\overline{g}_{\rm ex} = 0.5$ μS/mm^2 and $E_{\rm ex} = 0$. Also, the threshold and reset potentials for the model are $V_{\rm th} = -54$ mV and $V_{\rm reset} = -80$ mV. The excitatory conductance should satisfy the equation

$$\tau_{\rm ex}\frac{ds}{dt} = -s$$

   with $\tau_{\rm ex} = 10$ ms. In addition, every time there is a presynaptic action potential,

$$s \rightarrow s + 1 \, .$$

   Plot V(t) in one graph and the synaptic current, defined as,

$$I_{\rm ex} = \overline{g}_{\rm ex}s(V - E_{\rm ex}) \, ,$$

   in another. Trigger presynaptic action potentials at times 100, 200, 230, 300, 320, 400, and 410 ms. Explain what you see.

   Optional more advanced problem: Replace the synaptic conductance term $\overline{g}_{\rm ex}s(V - E_{\rm ex})$ with a synaptic current $\overline{g}_{\rm ex}s$ and consider the equation

$$\tau_{\rm m}\frac{dV}{dt} = E - V + \overline{g}_{\rm ex}s$$

   with s obeying its two equations above. Consider a single isolated presynaptic spike, and assume it is subthreshold for generating a postsynaptic spike. The peak for the resulting synaptic current is $\overline{g}_{\rm ex}$. Analytically calculate what the height of the peak of the resulting membrane potential depolarization is (the maximum of $V - E$) as a function of $\overline{g}_{\rm ex}$, $\tau_{\rm m}$ and $\tau_{\rm ex}$.

3. Construct an integrate-and-fire model responding to a "noisy" input representing the *in vivo* environment. This model is based on the equation

$$\tau_{\mathrm{m}} \frac{dV}{dt} = E - V(t) + \sqrt{2D}\eta(t) + I$$

with $\tau_{\mathrm{m}} = 10$ ms and $E = -56$ mV. The threshold and reset potentials for the model are $V_{\mathrm{th}} = -54$ mV and $V_{\mathrm{reset}} = -80$ mV. In your simulation, integrate the above equation using the Euler method with a suitably small Δt. At every time step, draw the value of $\eta(t)$ from a Gaussian (normal) distribution with mean 0 and standard deviation 1. Choose $D = \sigma_V^2 \tau_{\mathrm{m}}/\Delta t$, where σ_V, like I, is a parameter you will vary as discussed below.

a) Set $I = 0$ and turn off the spike generation mechanism in your model (by setting V_{th} to an extremely large value, for example). Plot the standard deviation of the membrane potential fluctuations that arise from different σ_V values in the range $0 \leq \sigma_V \leq 10$ mV.

b) Turn the spikes back on and plot the average firing rate of the neuron (defined by counting spikes over a sufficiently long time interval and dividing by the duration of that interval) as a function of I for σ_V =2, 6 and 10 mV. You may have to include negative I values in the range you consider to stop the neuron from firing. How does this differ from the firing-rate curve you determined in last week's assignment?

b) Set $I = 0$ and plot the average firing rate of the neuron as a function of σ_V for values in the range $0 \leq \sigma_V \leq 10$ mV. How does this differ from the firing-rate curve you determined in last week's assignment and the curve you obtained in b?