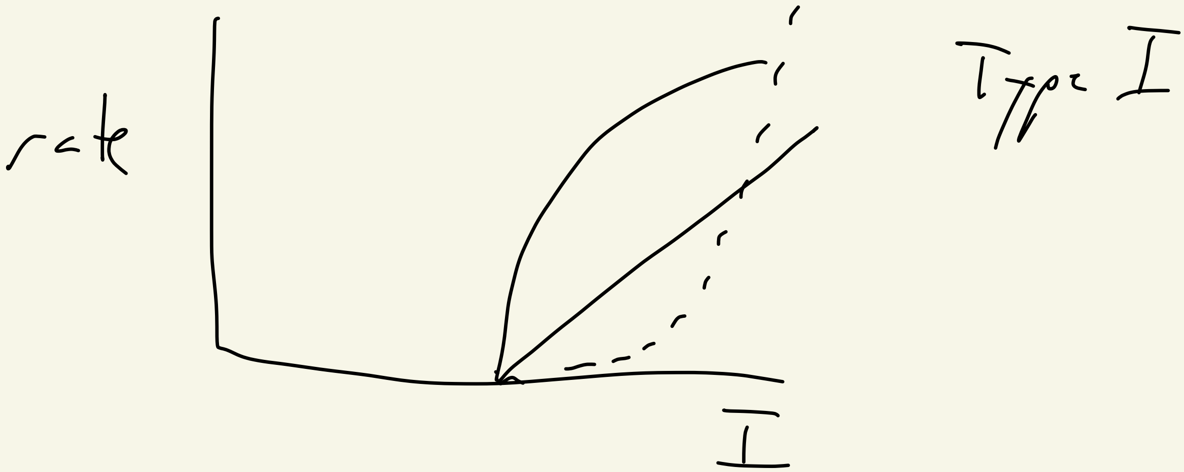
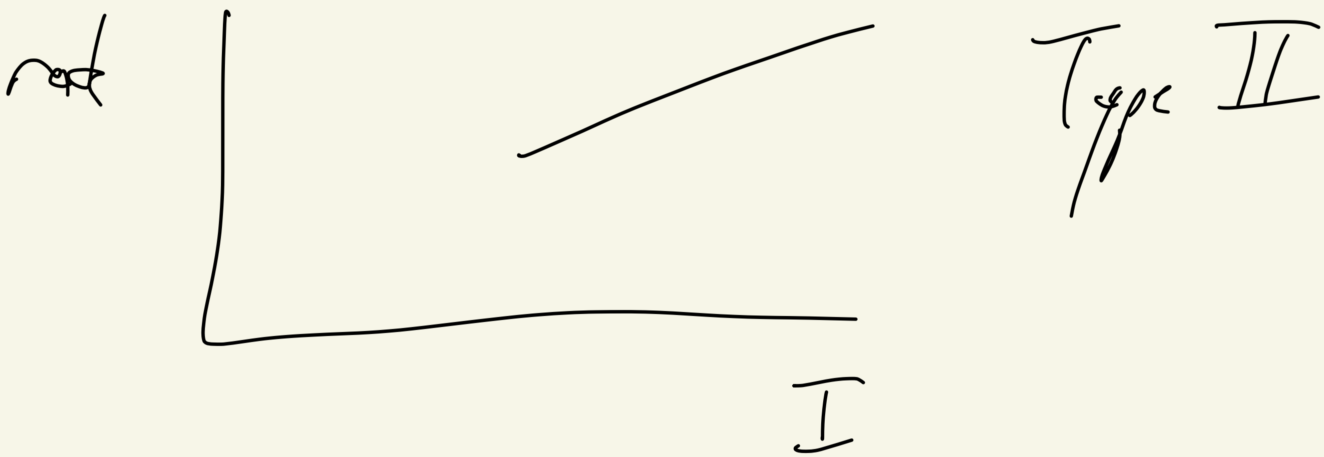
Firing-Rate Curves



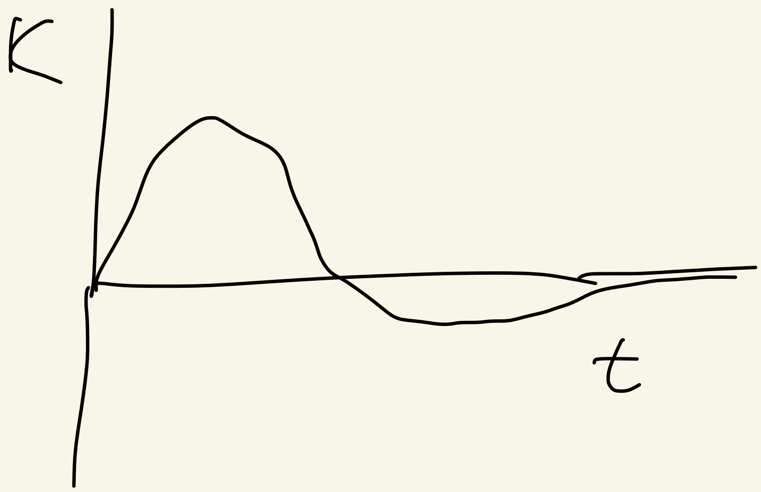


ReLu, Powerlaw, tanh, exp, $\ln(1 + \alpha e^{\beta I})$

$r = F(I)$ What is I?

In time:

$$I(t) = \int_{-\infty}^{t} dt' \, K(t-t') \, S(t')$$

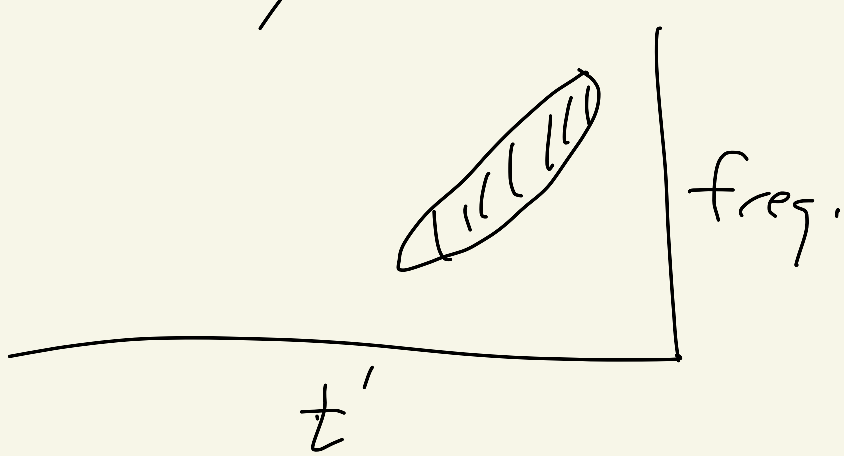


In "space"

$$I(t) = \int dx \int_{-\infty}^{t} dt' \, K(x, t-t') \, S(x, t')$$

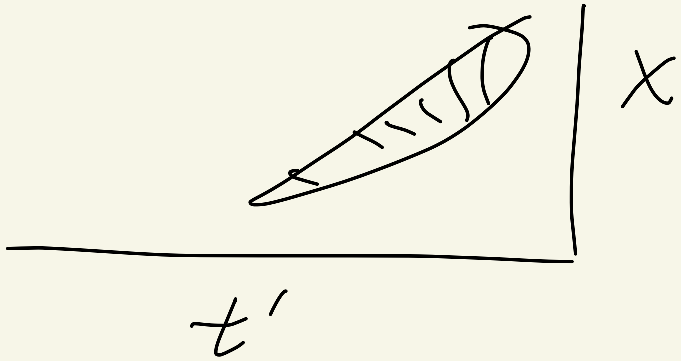
x = frequency

$(t=0)$



rising tone

visual

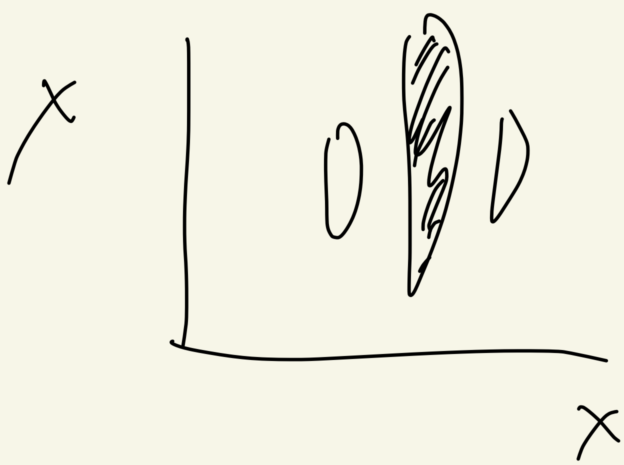


moving bar

Separable

$$K(x, t-t') = A(x)\, B(t-t')$$

$$x \to (x, y)$$



Specifically

$$A(x,y) = e^{-\frac{1}{2}\left(\frac{x-x_0}{\sigma_x}\right)^2} e^{-\frac{1}{2}\left(\frac{y-y_0}{\sigma_y}\right)^2} *$$

$$\cos(kx - \phi)$$

x_0, y_0 = R.F. location

σ_x, σ_y R.F. size

λ spatial wavelength

ϕ spatial phase

θ orientation preference

GLM Model

$$P(\text{data}) = P(\text{spike sequence} \mid r(t))$$

with $r(t) = r_0 \exp(I(t))$

Rate Models

$$r = F(I)$$

I is like $\bar{g}s$

$$\tau \frac{dI}{dt} = -I + \bar{g} r_{pre} \tau$$

$$\bar{g}\tau \rightarrow J \rightarrow J_{ij}$$

$$I \rightarrow x$$

$$\tau \frac{dx_i}{dt} = -x_i + \sum_j J_{ij} r_j$$

or

$$\boxed{\tau \frac{dx_i}{dt} = -x_i + \sum_j J_{ij} F(x_j)}$$

define $\quad x_i = \sum_j J_{ij} y_j$

$$\tau \cancel{\sum_j J_{ij}} \frac{dy_j}{dt} = - \cancel{\sum_j J_{ij}} x_j + \cancel{\sum_j J_{ij}} F\left(\sum_k J_{jk} y_k\right)$$

$$\boxed{\tau \frac{dy_i}{dt} = -y_i + F\left(\sum_j J_{ij} y_j\right)}$$

Stability

$$\tau \frac{dx_i}{dt} = -x_i + \sum_j J_{ij} F(x_j)$$

Suppose $F(\bar{x}_i) = 0$

$$x_i = \bar{x}_i + \delta x_i$$

$$\tau \frac{d\delta x_i}{dt} = -\delta x_i + \sum_j J_{ij} F'(\bar{x}_j)\delta x_j$$

often choose

$$F'(\bar{x}_j) = 1$$

so

$$\tau \frac{d\delta x_i}{dt} = -\delta x_i + \sum_j J_{ij}\, \delta x_j$$

stability involves eigenvalues of J_{ij}

Network Architecture

F.F.N. vs R.N.N.

Discrete vs Continuous Time

Learning