$$P_{spike} = r\Delta t$$

$$P_{nospike} = 1 - r\Delta t$$

$$P_n(T) = \frac{N!}{(N-n)!\,n!}(r\Delta t)^n (1 - r\Delta t)^{N-n}$$

with $T = N\Delta t$

$$\frac{N!}{(N-n)!} \simeq N^n = \left(\frac{T}{\Delta t}\right)^n$$

$$(1 - r\Delta t)^{N-n} \simeq e^{-Nr\Delta t} = e^{-rT}$$

so

$$P_n(T) = \frac{(rT)^n e^{-rT}}{n!}$$

$$\sum_{n=0}^{\infty} P_n(T) = e^{-rT} \sum_{n=0}^{\infty} \frac{(rT)^n}{n!} = e^{-rT} e^{rT} = 1$$

$$P_0(T) = e^{-rT}$$

$$\langle n \rangle = e^{-rT} \sum_{n=0}^{\infty} n \frac{(rT)^n}{n!}$$

$$= e^{-rT} \sum_{n=1}^{\infty} \frac{(rT)^n}{(n-1)!} = rTe^{-rT} \sum_{n=0}^{\infty} \frac{(rT)^n}{n!}$$

$$= rT$$

$$\langle n^2 \rangle = e^{-rT} \sum_{n=0}^{\infty} n^2 \frac{(rT)^n}{n!}$$

$$= e^{-rT} \sum_{n=1}^{\infty} \frac{n(rT)^n}{(n-1)!} =$$

$$= e^{-rT} rT \sum_{n=0}^{\infty} \frac{(n+1)(rT)^n}{n!}$$

$$= (rT)^2 + rT$$

$$\sigma_n^2 = \langle n^2 \rangle - \langle n \rangle^2 = rT$$

Fano Factor $= \dfrac{\sigma_n^2}{\langle n \rangle^2} = 1$

$$P\left(t \pm \tfrac{1}{2}\Delta t\right) = \rho(t)\Delta t$$

$$= r\Delta t\, e^{-rt}$$

$$\rho(t) = r e^{-rt}$$

$$\int_0^\infty dt\, \rho(t) = 1$$

$$r\int_0^\infty dt\, t\, e^{-rt} = -r\frac{d}{dr}\int_0^\infty dt\, e^{-rt}$$

$$= -r\frac{d}{dr}\frac{1}{r} = \frac{1}{r}$$

So $\quad \langle t_{isi} \rangle = \dfrac{1}{r}$

$$r\int_0^\infty dt\, t^2 e^{-rt} = r\frac{d^2}{dr^2}\int_0^\infty dt\, e^{-rt}$$

$$= r\frac{2}{r^3} = \frac{2}{r^2}$$

$$\sigma_{isi}^2 = \langle t_{isi}^2 \rangle - \langle t_{isi} \rangle^2 = \frac{1}{r^2}$$

$$C.V. = \frac{\sigma_{isi}}{\langle t_{isi} \rangle} = 1$$

$r(t)$

$$P(t_1, t_2 \ldots t_n) = \prod_i r(t_i)\, e^{-\int_0^T dt' r(t')}$$

$\rightarrow$ Maximum Likelihood