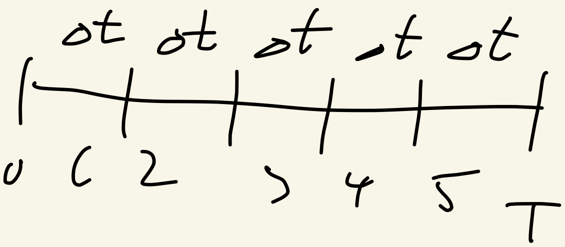
$$I = \sigma \eta \qquad \eta \sim G(0,1)$$

$\uparrow$ choose every Δt

average variance over a time T



$$i = 1, \dots \frac{T}{\Delta t}$$

$$I_{tot} = \frac{\Delta t}{T} \sigma (\eta_1 + \eta_2 + \cdots)$$

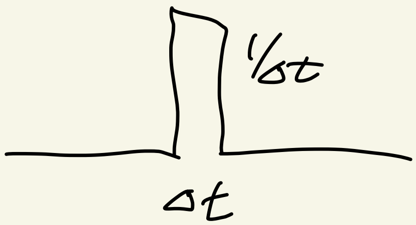
$$\sigma^2_{tot} = \left(\frac{\Delta t}{T}\right)^2 \left(\frac{T}{\Delta t}\right) \sigma^2 = \frac{\Delta t}{T} \sigma^2 \to 0!$$

so

$$\boxed{\sigma = \sigma_v \sqrt{\frac{2\tau}{\Delta t}}}$$

$$\langle I(t_1) I(t_2) \rangle = 0 \quad \text{if } t_1 \neq t_2$$

$$\langle I(t_1) I(t_2) \rangle = \sigma^2 = \frac{2\sigma_v^2 \tau}{\Delta t}$$



$$= \delta(t_1 - t_2)$$

$$\int dt\, \delta(t) = 1 \qquad \int dt'\, f(t')\,\delta(t-t') = f(t)$$

$$\langle I(t_1)\, I(t_2) \rangle = 2\sigma_V^2 \tau\, \delta(t_1 - t_2)$$

$$\tau \frac{dV}{dt} = E - V + I \qquad V = E + x$$

$$\tau \frac{dx}{dt} = -x + I$$

$$x(t) = \frac{1}{\tau}\int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau}\, I(t')$$

$$\langle x \rangle = \frac{1}{\tau}\int_{-\infty}^{t} dt'\, e^{-(t-t')/\tau}\, \langle I \rangle = 0$$

$$\sigma_x^2 = \langle x^2(t) \rangle = \frac{1}{\tau^2}\int_{-\infty}^{t} dt' \int_{-\infty}^{t} dt''\, e^{-(t-t')/\tau}\, e^{-(t-t'')/\tau}$$

$$\langle I(t)\, I(t') \rangle$$

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$$\sigma_x^2 = \langle x^2(t)\rangle = \frac{2\sigma_v^2 \tau}{\tau^2}\int_{-\infty}^{t} dt' \int_{-\infty}^{t} dt'' \, e^{-(2t-t'-t'')/\tau} \, \delta(t-t')$$

$$= \frac{2\sigma_v^2}{\tau}\int_{-\infty}^{t} dt' \, e^{-2(t-t')/\tau}$$

$$= \frac{2\sigma_v^2}{\tau}\,\frac{\tau}{2} = \sigma_v^2$$