Volt = Ampere * Ohm

Ampere * second = Coulomb

Farad * Volt = Coulomb

Farad * Volt/second = Ampere

Farad * Ohm = second

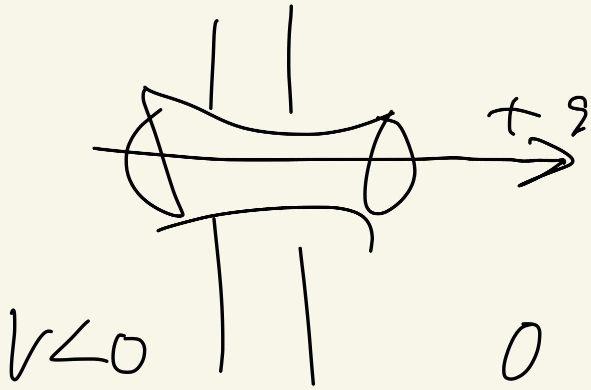
$r_L = 1 k\Omega - mm$

$r_m = 1 M\Omega - mm^2$

$c_m = 10 nF/mm^2$

$A \approx 0.1\ mm^2$ mV, nA, MΩ, nF

1)

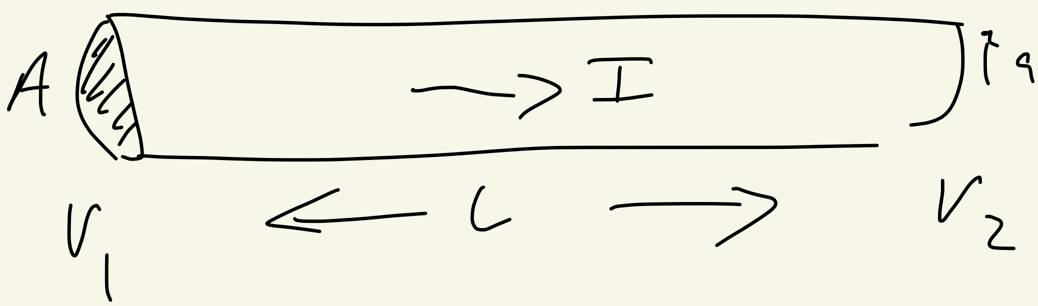


K^+ for example

$$E = q|V| \sim k_B T$$

$$|V| = \frac{k_B T}{q} = V_T \simeq 25\,mV$$

2)



$$V_1 - V_2 = I R_L$$

$$R_L \sim \frac{L}{A} = \frac{r_L L}{A}$$

$$r_L = 1\,K\Omega\text{-}mm$$

example

$$L = 100\,\mu m = 0.1\,mm$$

$$a = 2\,\mu m = 2 \times 10^{-3}\,mm$$

$A = \pi a^2 \quad R_L = \dfrac{(10^3 \Omega\text{-mm})(0.1\text{ mm})}{\pi \cdot 4 \times 10^{-6}\text{ mm}^2}$ ②

$$= \frac{10^2 \Omega}{1.2 \times 10^{-5}} = \frac{10}{1.2} \times 10^6 \Omega$$

$$\sim 8\text{ M}\Omega$$

$$V_1 - V_2 = 8\text{ mV}$$

$$I = \frac{8\text{mV}}{8\text{M}\Omega} = 1\text{nA}$$

example 2

$$g_L = \frac{1}{R_L} = \frac{A}{r_L L}$$

channel

$$L = 6\text{nM} = 6 \times 10^{-6}\text{ mm}$$

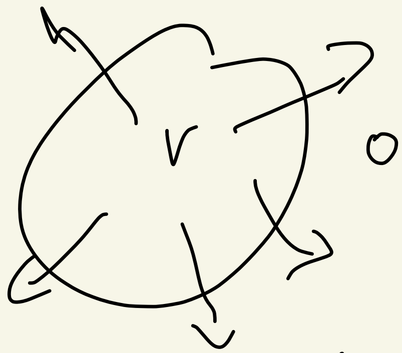
$$A = 0.15\text{ nm}^2 = 15 \times 10^{-14}\text{ mm}^2$$

$$g_c = \frac{15 \times 10^{-14}\, mm^2}{6 \times 10^{-6}\, mm \cdot 10^3\, \Omega\, mm} = 2.5 \times 10^{-11}\, S$$

$$= 25 \times 10^{-12}\, S = 25\, pS$$

3)

a)



$$I_m = g_m (V - E)$$

E = resting potential $\simeq -70\, mV$

$$\Delta V = V - E \qquad I_m = g_m \Delta V$$

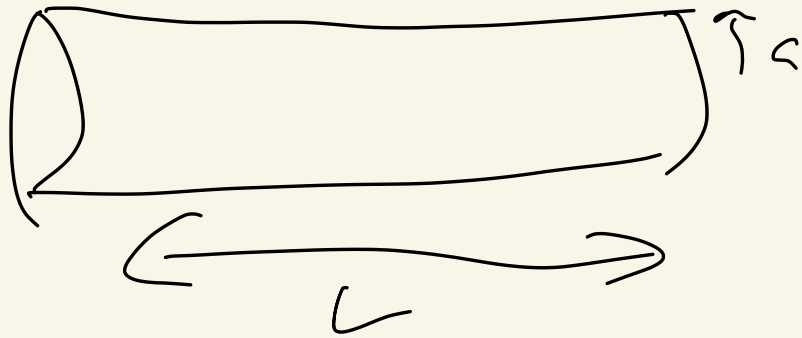
$$g_m \sim A_m \qquad R_m = \frac{1}{g_m} = \frac{r_m}{A}$$

$$r_m = 1\, M\Omega\, mm^2$$
$$A_m = 0.1\, mm^2$$
$$R_m = 10\, M\Omega$$

for $\Delta V = 10\, mV$

$$I = \frac{10\, mV}{10\, M\Omega} = 1\, nA$$

b)



$$R_L = \frac{r_L L}{\pi a^2} \qquad R_m = \frac{r_m}{2\pi a L}$$

what L makes

$$R_L = R_m$$

$$\frac{r_L L}{\pi a^2} = \frac{r_m}{2\pi a L}$$

$$L^2 = \frac{r_m}{r_L} \frac{\pi a^2}{2\pi a} = \frac{a r_m}{2 r_L}$$

$$L = \lambda = \sqrt{\frac{a r_m}{2 r_L}} \sim 1\,mm$$

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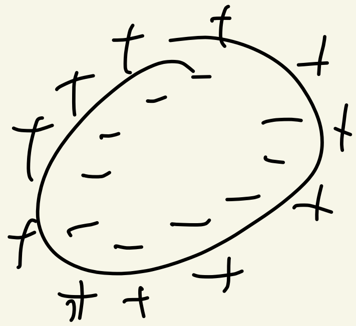
$$\frac{2 \times 10^{-3} \cdot 10^{6}}{2 \cdot 10^{3}} = 1$$

4) b



$Q = CV$

$C = c_m A \qquad c_m = 10\,nF/mm^2$

$C = 10\,nF \times 0.1 = 1\,nF$

$Q = 70\,mV \cdot 1\,nF = 7 \times 10^{-11}\,C$

$\approx 10^{9}$ charges

$= 10^{-5}$ of charges in cell

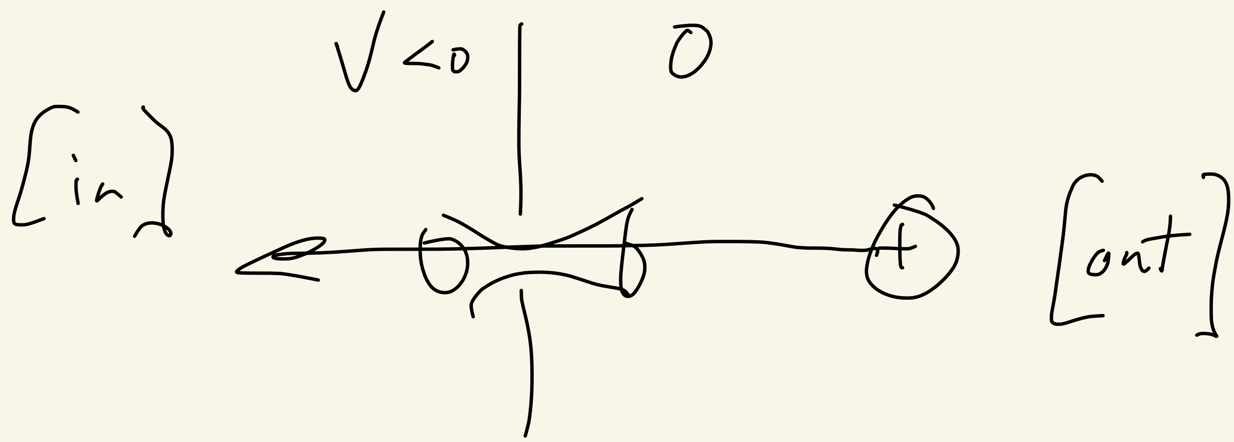
$Q = 0.7\,nA \cdot 100\,ms = 7 \times 10^{-11}\,C$

$$\boxed{\frac{dQ}{dt} = I = C\frac{dV}{dt}}$$

I = positive <u>inward</u>

$$10^{-9} F \frac{10 mV}{10 mS} = 10^{-9} A = 1 nA$$

100 times greater for A.P.



At equilibrium

$$\frac{[out]}{[in]} = e^{-q|V|/k_B T}$$

by definition, this voltage is E

so

$$e^{E/V_T} = \frac{[out]}{[in]}$$

with

$$V_T = \frac{k_B T}{q}$$

$$E = V_T \ln\left(\frac{[out]}{[in]}\right)$$ Nernst Potential

for multiply charged ions

$$E = \frac{V_T}{z} \ln\left(\frac{[out]}{[in]}\right)$$

	ion	E	
outward	K^+	$-80\,mV$	$[in] > [out]$
inward	Na^+	$+50\,mV$	$[in] < [out]$
inward	Ca^{++}	$+200\,mV$	$[in] << [out]$
~	Cl^-	$-60\,mV$	$[in] < [out]$
	exc.	$0\,mV$	
	inh.	E_{K^+} or E_{Cl^-}	

also ion pumps – Na-K exchanger

$I = g_m (V - E)$ is positive <u>outward</u>

so $$\boxed{C \frac{dV}{dt} = -g_m (V-E) + I_{ext}}$$

5) E?

$$I_m = g_K (V - E_K) + g_{Na} (V - E_{Na}) + \cdots$$

$$g_m = g_K + g_{Na} + \cdots$$

$$E = \frac{g_K E_K + g_{Na} E_{Na} + \cdots}{g_K + g_{Na} + \cdots}$$

6) Passive neuron model

$$C\frac{dV}{dt} = -g_m(V-E) + I_{ext}$$

$$C = c_m A \qquad g_m = \frac{A}{r_m}$$

$$c_m r_m \frac{dV}{dt} = E - V + R_m I_{ext}$$

$$c_m r_m = \tau = 10\times 10^{-9}\,F \cdot 1\times 10^{6}\,\Omega$$

$$= 10\,ms$$

$$\frac{\lambda}{\tau} = \frac{1\,mm}{10\,ms} = \frac{0.1\,mm}{ms} = \frac{0.1\,m}{s}$$

$a \to 2 \to 200\,\mu m$ ~ 1 m/s   big axons

~ 100 m/s   mylination

$$\tau \frac{dV}{dt} = E - V + R_m I_{ext}$$

$$R_m I_{ext} = I$$

$$\tau \frac{dV}{dt} = E + I - V$$

a) $I = \text{constant}$

$$E + I = V_\infty$$

$$\tau \frac{dV}{dt} = V_\infty - V$$

$$V = V_\infty + x$$

$$\tau \frac{dx}{dt} = -x$$

$$x(t) = x(0)\, e^{-t/\tau}$$

or

$$x(t_2) = x(t_1)\, e^{-(t_2 - t_1)/\tau}$$

or

$$x(t + \Delta t) = x(t)\, e^{-\frac{\Delta t}{\tau}}$$

$$x = V - V_{\infty}$$

so

$$V(t) = V_{\infty} + (V(0) - V_{\infty})e^{-t/\tau}$$

or

$$V(t_2) = V_{\infty} + (V(t_1) - V_{\infty})e^{-(t_2 - t_1)/\tau}$$

or

$$V(t + \Delta t) = V_{\infty} + (V(t) - V_{\infty})e^{-\frac{\Delta t}{\tau}}$$

7) Integrate + Fire Model

$$\tau \frac{dV}{dt} = E - V + I$$

when $V = V_{th}$ spike

and $V \to V_{reset}$

a) Firing rate for constant I

$$\underline{E + I \geq V_{th}}$$

at $t = 0$

$$V = V_{reset}$$

when is

$$V(t) = V_{th}$$

$$V_{\infty} = E + I$$

$$V_{th} = V_\infty + (V_{reset} - V_\infty)e^{-t/\tau}$$

$$e^{t/\tau} = \frac{V_{reset} - V_\infty}{V_{th} - V_\infty} = \frac{V_\infty - V_{reset}}{V_\infty - V_{th}}$$

$$t = \tau \ln\left(\frac{V_\infty - V_{reset}}{V_\infty - V_{th}}\right)$$

$$r = \frac{1}{t}$$

8) Other conductances

$$c\frac{dV}{dt} = -g_L(V - E_L) - g_1(V - E_1) - \cdots$$

$$- g_s(V - E_s) + \cdots$$

$$g \to g(t)$$

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b) $I(t)$

$$V = E + x$$

$$\tau \frac{dx}{dt} = -x + I(t)$$

$$x = e^{-t/\tau} y$$

$$\tau \frac{dx}{dt} = -x + e^{-t/\tau} \frac{dy}{dt} = -x + I$$

$$\tau \frac{dy}{dt} = e^{t/\tau} I(t)$$

$$y(t) = \frac{1}{\tau} \int_0^t dt' \, e^{t/\tau} I(t')$$

$$x(t) = \frac{1}{\tau} e^{-t/\tau} \int_0^t dt' \, e^{t'/\tau} I(t')$$

$$= \frac{1}{\tau} \int_0^t dt' \, e^{-(t-t')/\tau} I(t')$$

$$= \int_0^t dt' \, K(t-t') \, I(t')$$

$$K(t-t') = \frac{1}{\tau} e^{-(t-t')/\tau}$$

another way to write it

$$\frac{1}{\tau}\int_0^t dt' e^{-(t-t')/\tau} I(t')$$

$$t'' = t - t'$$

$$t' = t - t''$$

$$\frac{1}{\tau}\int_0^t dt'' e^{-t''/\tau} I(t-t'')$$