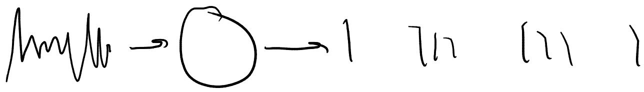
Estimating receptive fields



1) Spike-triggered average
2) Reverse correlation
3) Max. likelihood.

} Dayan & Abbott Ch. 1, 2

1) Let $s(t)$ be stimulus and $\{t_i\}$ be spike times

$$\text{STA} \quad C(\tau) = E\left[\frac{1}{n}\sum_{i=1}^{n} s(t_i - \tau)\right]$$

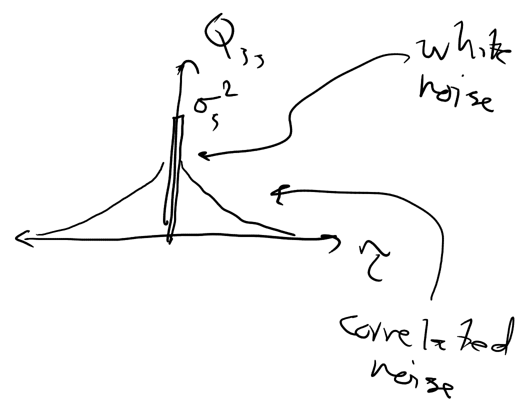
2) Let $r(t)$ be avg firing rate over trials, rate-stim. correlation fn. is:

$$Q_{rs}(\tau) = \frac{1}{T}\int_0^T dt\, r(t)\, s(t+\tau) = E[r]\, C(-\tau)$$

(replace sum in STA w/ integral of δ-functions)

Stim-stim. correlation fn. is:

$$Q_{ss}(\tau) = \frac{1}{T}\int_0^T dt\, s(t)\, s(t+\tau)$$



Predicted neural activity (linear):

$$r_{est} = r_0 + \int_0^\infty d\tau\, D(\tau)\, s(t-\tau)\,.$$ What is $D(\tau)$?

Minimize $E = \frac{1}{T}\int_0^t dt \left(r_{est}(t) - r(t)\right)^2$

Functional derivative wrt $D = 0 \Longrightarrow$

$$\underbrace{\int_0^\infty d\tau' \, Q_{ss}(\tau - \tau') \, D(\tau')} = Q_{rs}(-\tau)$$

"reverse correlation"

If white noise, only $\tau = \tau'$ contributes

$= \sigma_s^2 \, D(\tau)$

STA

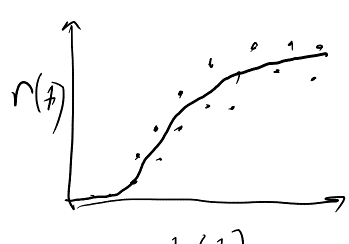
$$\Rightarrow D(\tau) = \frac{1}{\sigma_s^2} Q_{rs}(-\tau) = \frac{E[r]}{\sigma_s^2} C(\tau)$$

Best linear estimate of r.

What if $r_{est} = r_0 + F(L(t))$, $L(t) = \int_0^t d\tau \, D(\tau) \, s(t - \tau)$?

Bussgang's theorem: If s is white Gaussian noise, the above $D(\tau)$ is still optimal.

What is F?



Reverse correlation:

1) Apply Gaussian white noise stim.
2) Calculate optimal linear filter
3) Determine nonlinearity

Generalized linear models & MLE (J. Pillow, L. Paninski)

Linear Poisson neuron: $\lambda = \theta s$, $y \sim \text{Pois}(\lambda)$

rate

spk count

Recall Poisson distribution, $p(x|\lambda) = \frac{1}{x!}\lambda^x e^{-\lambda}$

$$P_\theta(x|s) = \frac{1}{x!}(\theta s)^x e^{-(\theta s)}$$

Suppose we have data X, S, $X = \{x_i\}$, $S = \{s_i\}$

$$p_\theta(X|S) = \prod_i p_\theta(x_i|s_i) \quad \text{(conditional independence)}$$

Find θ to max $p_\theta(X|S) \iff \max \log p_\theta(X|S)$.

$$\log p_\theta(X|S) = \log \prod_i p_\theta(x_i|s_i) = \sum_i \log p_\theta(x_i|s_i).$$

$$\log p_\theta(x_i|s_i) = \underline{-\log(x_i!)} + x_i(\log\theta + \underline{\log s_i}) - \theta s_i$$

don't depend on θ.

$$\log p_\theta(X|S) = \sum_i x_i \log\theta - \theta s_i + C$$

$$= \log\theta\left[\sum_i x_i\right] - \theta\left[\sum_i s_i\right] + C$$

To max. wrt θ, $\frac{d}{d\theta} P_\theta(X|s) = 0$

$$\Rightarrow 0 = \frac{1}{\theta} \sum_i x_i - \sum_i s_i$$

$$\Rightarrow \theta_{ML} = \sum_i x_i \Big/ \sum_i s_i$$

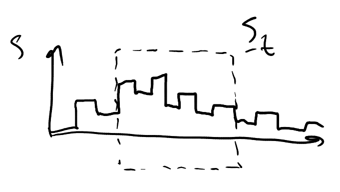
GLMs:

Linear-nonlinear-Poisson model:

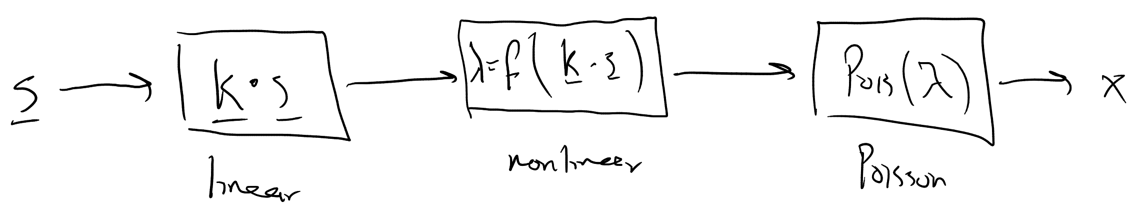
$$\lambda = f(\underline{k} \cdot \underline{s}), \qquad x = \mathrm{Pois}(\lambda).$$

nonlinearity — linear kernel — stimulus

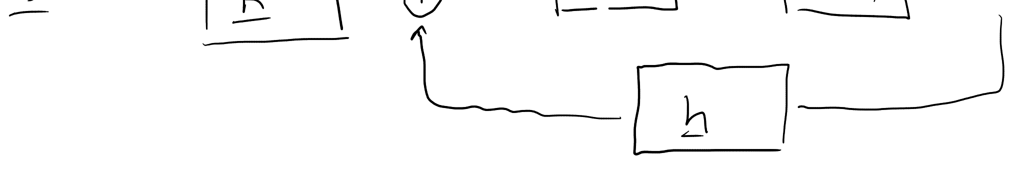
$\underline{s}$ can be vector of times & stim. dimensions.



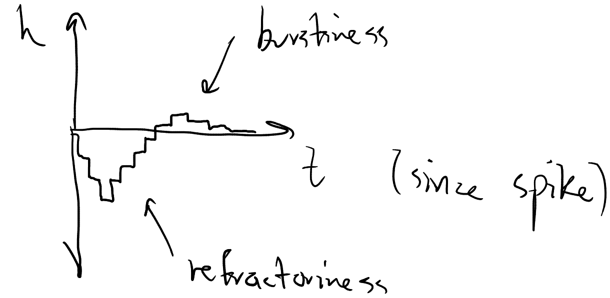
HW: Compute θ_{ML} for LNP model.



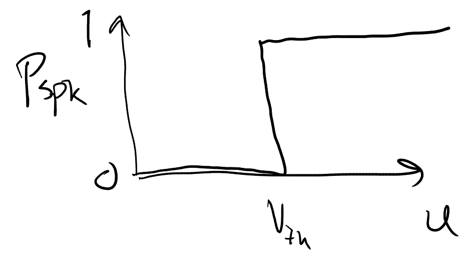
Spike-history dependence (bursting, refractoriness):

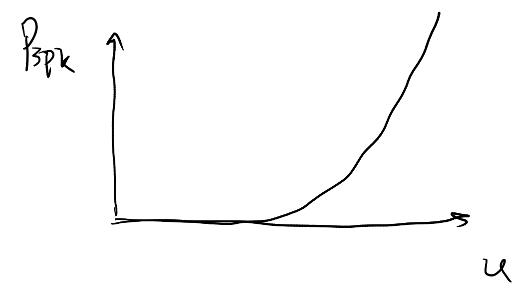


$\underline{h}$: post-spike filter:



Typically, $f(u)=e^u$. Can be thought as "soft-threshold" integrate & fire neuron:





Multiple GLMs can be coupled to infer $\underline{k}_i$, $\underline{h}_i$, and $\underline{W}_{ij}$ simultaneously. Inferred coupling $\neq$ synaptic coupling.

Combining (Macke, Büsing, Cunningham, Yu, Shenoy, Sahani 2011)

Ex: Poisson LDS:

$$\underline{z}_t \sim A\underline{z}_{t-1} + \underline{\eta}$$

$$x_{t,i} \sim \text{Pois}\left(\exp\left(C\underline{z}_t + H\underline{x}^{hist} + b\right)_i\right)$$