Assignment 6

1. Suppose we have a firing-rate network as in Sompolinsky, Crisanti & Sommers (1988) that evolves according to:

$$\dot{\mathbf{x}}(t) = -\mathbf{x}(t) + \mathbf{J}\phi(\mathbf{x}(t)), \tag{1}$$

where $\mathbf{J}$ is an $N \times N$ matrix of connections with $N = 500$ and each element of $\mathbf{J}$ is drawn from a normal distribution with standard deviation $g/\sqrt{N}$. Further assume that ϕ is the tanh function, applied to each element of the vector $\mathbf{x}$.

a) As discussed in class, the network dynamics linearized around the fixed point $\mathbf{x} = \mathbf{0}$ are:

$$\dot{\mathbf{x}}(t) = (-\mathbf{I} + \mathbf{J})\mathbf{x}(t). \tag{2}$$

Compute the eigenvalues of the matrix $-\mathbf{I} + \mathbf{J}$ numerically and make a scatter plot of their real and imaginary parts in the complex plane, for $g = 0.9$ and $g = 1.1$.

b) Simulate the *linearized* network (2) for 100 time units and plot the activity of 5 example neurons for $g = 0.9$, $g = 1.1$. Choose the initial condition $\mathbf{x}(0)$ to be a vector of elements each drawn from a standard normal distribution.

c) Do the same as part 2 but for 500 time units of the nonlinear network's dynamics. Also plot the behavior for $g = 2$.

2. From Dayan & Abbott: Simulate the responses of four interneurons in the cercal system of the cricket and check the accuracy of a simple population decoding scheme. In this system, there are four interneurons whose firing rate responses are dependent on the wind direction θ.

For a true wind direction θ the average firing rates of the four interneurons should be generated as $\mathrm{E}[r_i] = 50\text{ Hz} \cdot f(\cos(\theta - \theta_i))$, where f is rectified-linear ($f(x) = x$ if $x > 0$, $f(x) = 0$ otherwise), and $\theta_i = \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ for $i = 1, 2, 3, 4$. The actual rates, r_i, are then obtained by adding to these mean rates a random number chosen from a Gaussian distribution with zero mean and a standard deviation of 5 Hz (set any rates that come out negative to zero). Assume the noise is independent across neurons.

From these rates, construct the x and y components of the population vector:

$$x = \sum_{i=1}^{4} r_i \cos(\theta_i), \;\; y = \sum_{i=1}^{4} r_i \sin(\theta_i) \tag{3}$$

and, from the direction of this vector, compute an estimate θ_{est} of the wind direction. Average the squared difference $(\theta - \theta_{\text{est}})^2$ over 1000 trials. The square root of of this quantity is the error. Plot the error as a function of θ over the range $[-90°, 90°]$.