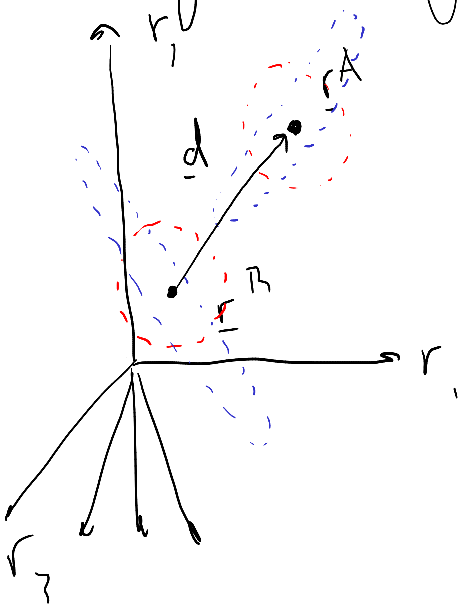
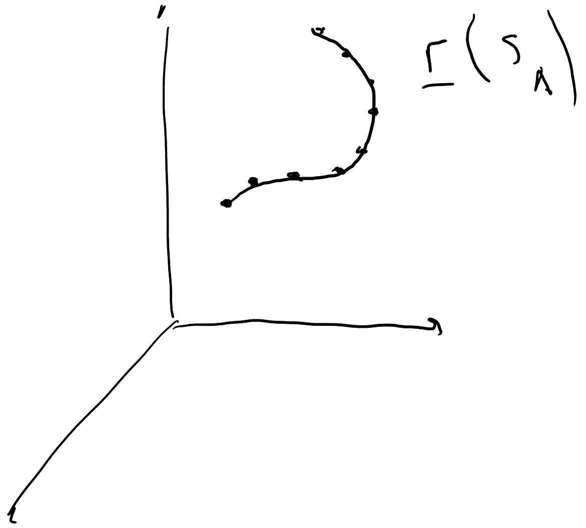
Encoding & decoding



Discrete stimuli



Continuous stimuli

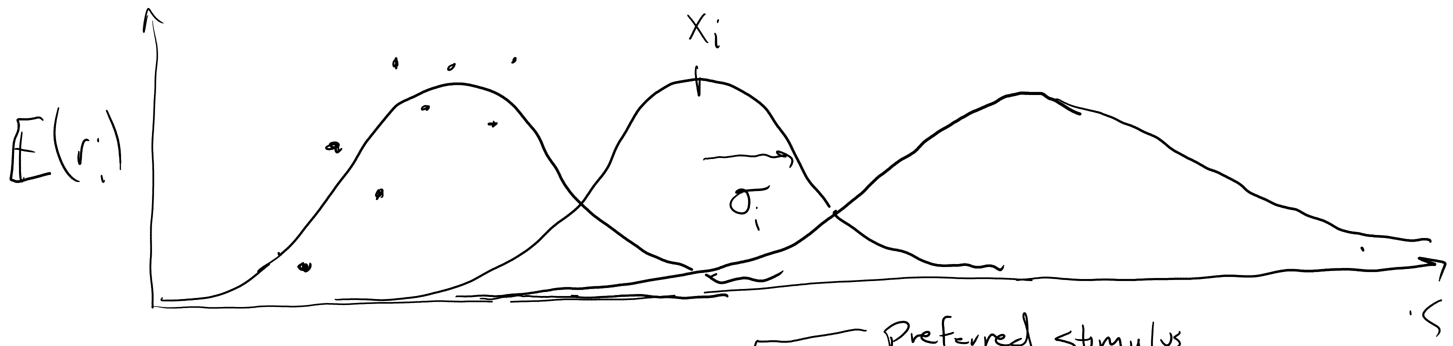
Discrete case:

1) Coding direction: $\underline{r}^A - \underline{r}^B = \underline{d}$
2) Nearest neighbors
3) Fisher's linear discriminant: Gaussian noise Σ_A, Σ_B.
4) Binary classifiers (later).
5) MLE

Continuous case:

1) Linear regression
2) MLE

Population decoding: Example



$$\lambda_i = r_{max} \exp\left(\frac{-(s - s_i)^2}{2\sigma_i^2} \right)$$

preferred stimulus

tuning width

$r_i \sim \mathrm{Pois}(\lambda_i)$

If independent,

$$P(\underline{r}|s) = \prod_i P(r_i|s)$$

$$= \prod_i \frac{\lambda_i^{r_i}}{r_i!} e^{-\lambda_i}$$

$$\log P(\underline{r}|s) = \sum_i r_i \log \lambda_i(s) - \log r_i! - \lambda_i(s)$$

Simplifying assumption: $\sum_i \lambda_i(s) \approx \text{const.}$

Maximize $\sum_i r_i \log \lambda_i(s) = \sum_i r_i \frac{-(s-s_i)^2}{2\sigma_i^2} + C$
$+C$

Derivative wrt $s = 0 \Longrightarrow$

$$0 = \sum_i r_i \frac{-\cancel{2}(s-s_i)}{\cancel{2}\sigma_i^2}$$

$$s_{ML} = \sum_i r_i s_i/\sigma_i^2 \Big/ \sum_i r_i/\sigma_i^2$$

Preferred stimulus of neuron i weighted by r_i/σ_i^2

If σ_i constant, reduces to weighted sum

$$s_{ML} = \sum_i r_i s_i \Big/ \sum r_i$$

Maximum likelihood estimate.

Bayes rule: $P(s|\underline{r}) = \dfrac{P(\underline{r}|s)P(s)}{P(\underline{r})}$

Maximum a posteriori estimate: maximize

$P(s|\underline{r})$ given prior $P(s) \Longleftrightarrow$

Maximize $P(\underline{r}|s)P(s)$. If $P(s) = \text{constant}$

(flat prior), MAP estimate = MLE

What about correlated case? Can no longer factorize likelihood.

Population coding, correlations

Spike count correlations:

$$\rho = \frac{\mathrm{Cov}(r_1, r_2)}{\sqrt{\mathrm{Var}(r_1)\,\mathrm{Var}(r_2)}}$$

Stimulus correlation:

$$\text{Let } \bar{r}_1(s) = E[r_1 | s]$$

$$\rho_{stim} = \frac{\mathrm{Cov}_s(\bar{r}_1(s), \bar{r}_2(s))}{\sqrt{\mathrm{Var}_s(\bar{r}_1(s))\,\mathrm{Var}_s(\bar{r}_2(s))}}$$

"similarity in tuning"

Noise correlation:

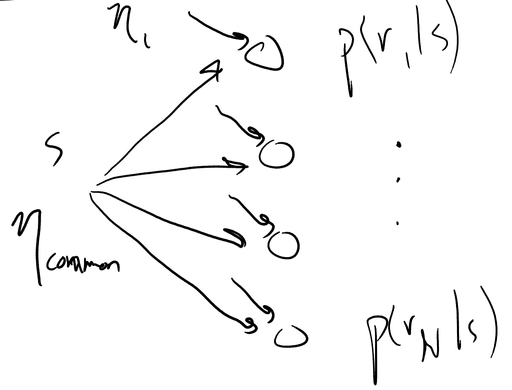
$$\rho_{sc}(s) = \frac{\mathrm{Cov}(r_1, r_2 | s)}{\sqrt{\mathrm{Var}(r_1|s)\,\mathrm{Var}(r_2|s)}}$$

"trial-to-trial covariability"

Often assumed $\rho_{sc} = E_s[\rho_{sc}(s)]$.

Dependence on s: "stimulus-dependent noise correlations"

Implications for stimulus decoding (Zohary, Shadlen, Newsome 1994)



$$p(r_i|s) = p(r|s)$$

$$R = \sum_i r_i$$

$$E[R|s] = N\,E[r|s] \qquad \text{"signal"}$$

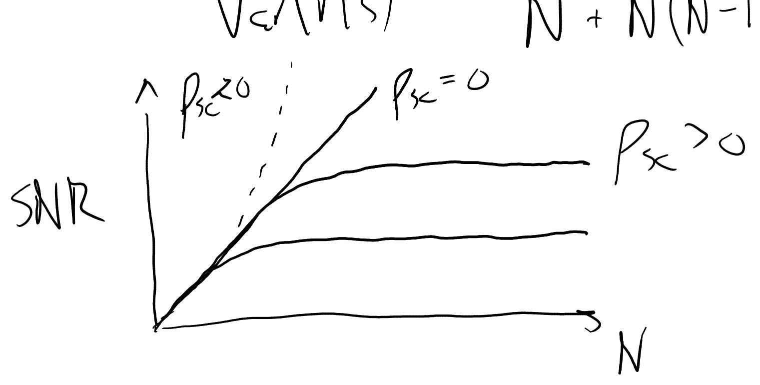
$$\mathrm{Var}(R|s) = \mathrm{Var}\left(\sum_i r_i \,\Big|\, s\right) = E\left[\sum_i (r_i - E[r_i])^2 + \sum_{i \neq j} (r_i - E[r_i])(r_j - E[r_j])\right]$$

conditioned on s

$$= N\,\mathrm{Var}(r|s) + N(N-1)\,\mathrm{Cov}(r_i, r_j|s)$$

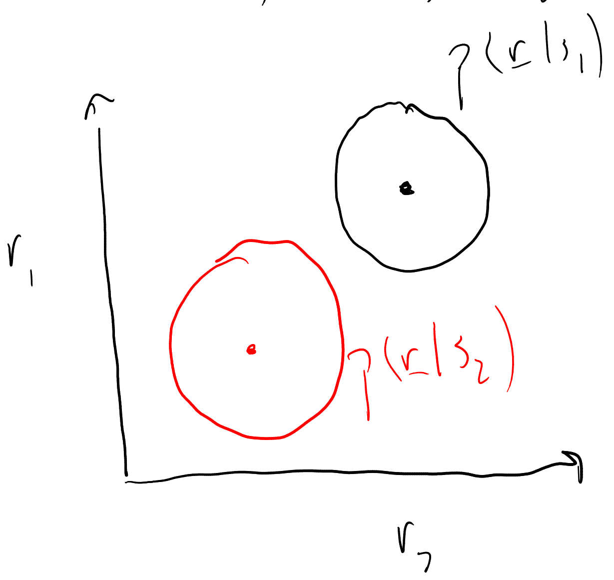
$$= \mathrm{Var}(r|s)\left[N + N(N-1)\rho_{sc}\right]$$

$$SNR = \frac{E^2[r|s]}{\mathrm{Var}(r|s)} \cdot \frac{N^2}{N + N(N-1)\rho_{sc}}$$



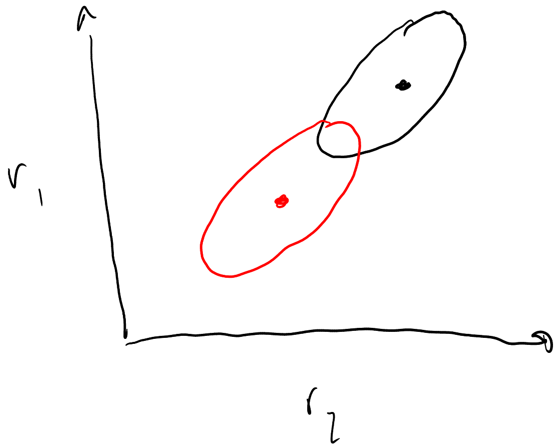
$\rho_{sc} \sim 1/N$ detrimental

Averbeck, Latham, Pouget Nat. Rev. Neurosci 2006



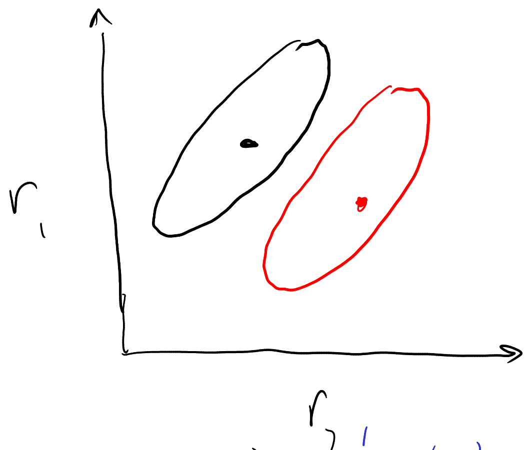
$\rho_{stim} > 0$

$\rho_{sc} = 0$



$\rho_{stim} > 0$

$\rho_{sc} > 0$



$\rho_{stim} < 0$

$\rho_{sc} > 0$