Assignment 7

G4360 Introduction to Theoretical Neuroscience
DUE: April 8, 2019 11:59 pm

1 Variability of Neural Traces [1]

In this problem we are going to study the effect of the stimulus onset on the variability of the neural responses using PCA. Neural variability is often treated as random noise, generated either by other brain areas, or by stochastic processes within the circuitry being studied. We call such sources of variability "external" to stress the independence of this form of noise from activity driven by the stimulus. Variability can also be generated internally by the same network dynamics that generates responses to a stimulus. How can we distinguish between external and internal sources of response variability? Our goal here is to show that the internal sources of variability interact nonlinearly with stimulus-induced activity, and this interaction yields a suppression of noise in the evoked state.

We use a model described by [1] where the neurons are described by firing-rates, they do not fire individual action potentials. These networks are more attractive than spiking networks because analyzing them is much easier. Moreover, as long as there is no large-scale synchronization of the spikes in a network, the rate model can describe the network dynamics accurately. We consider a network of N interconnected neurons, with neuron i characterized by an activation variable x_i satisfying

$$\tau \frac{dx_i}{dt} = -x_i + g \sum_{i=1}^{N} J_{ij} r_j + I_i \tag{1}$$

The time constant τ is set to 10 ms. The recurrent synaptic weight matrix $\boldsymbol{J}$ has element J_{ij} describing the connection from presynaptic neuron j to postsynaptic neuron i. The input term, I_i denotes the external input for neuron i.

The firing rate of neuron i is given by $r_i = R_0 + \Phi(x_i)$ where:

$$\Phi(x_i) = \begin{cases} R_0 \tanh(\frac{x}{R_0}) & x \le 0 \\ (R_{\max} - R_0) \tanh(\frac{x}{R_{\max} - R_0}) & x > 0 \end{cases}$$

Here, R_0 is the background firing rate (the firing rate when $x = 0$), and $R_{\max}$ is the maximum firing rate. This function allows us to specify independently the maximum firing rate, $R_{\max}$, and the background rate, R_0, and set them to reasonable values, while retaining the general form of the commonly used tanh function.

1.1 [Optional]

To facilitate comparison with experimental data in a variety of systems, we report all responses relative to $R_{\max}$. Similarly, we would like to find a proper scaling for the input current I. One way of finding a proper scale for the input current is to find the value $I_{\frac{1}{2}}$ which is the current that is required to drive a neuron in

[1] The model, main ideas, and parts of text in this problem are taken from [1]

the network to half of its maximal firing rate (we find $I_{\frac{1}{2}}$ by empirical examination rather than mathematical derivation). The code for simulating the neural traces using the model in equation 1 is provided. Use this code and plot the external input I as a function of mean($\frac{r}{R_{\max}}$) and find $I_{\frac{1}{2}}$.

1.2

We construct the connection matrix $\boldsymbol{J}$ of the model by choosing elements of the synaptic weight matrix independently and randomly from a Gaussian distribution with zero mean and variance $\frac{1}{N}$. The parameter g controls the strength of the synaptic connections in the model, but because these strengths are chosen from a distribution with zero mean and nonzero variance, g actually controls the size of the standard deviation of the synaptic strengths (note that the weights are fixed for a given network and they don't fluctuate, in fact g controls the standard deviation of the distribution over all the weights sampled independently). Without any input ($I_i = 0$ for all i) and for large networks (large N), two spontaneous patterns of activity are seen. If $g < g_0$, the network is in a trivial state in which $x_i = 0$ and $r_i = R_0$ for all neurons (all i). The case $g > g_0$ is more interesting in that the spontaneous activity of the network is chaotic, meaning that it is irregular, non-repeating and highly sensitive to initial conditions. Try different values of $g \in \{1, 1.5, 2, 2.5\}$ and for each g plot the traces of 10 neurons chosen randomly. Which g makes the network chaotic?

1.3

Simulate the neural activities for 2 seconds and inject a step input into the network after 1 second as described below. Collect all the neural responses into an $N \times T$ matrix $\boldsymbol{r}$ (where N is the number of simulated neurons and T is the number of time points). Run PCA on this matrix treating the columns of the $\boldsymbol{r}$ matrix as points in $\mathbb{R}^N$. Plot the percentage of variance explained as a function of the number of PCs. Do this separately for evoked ($t \geq 1$) and spontaneous ($t < 1$) activity (since 1.1 is optional, if you haven't solved it use $I_{\frac{1}{2}}$=1 for this problem). Describe your observations and conclusions.

$$I = \begin{cases} I_{\text{step}} & t \geq 1 \\ 0 & t < 1 \end{cases} \qquad I_{\text{step}} \in \{I_{\frac{1}{2}}, 2I_{\frac{1}{2}}, 4I_{\frac{1}{2}}\}$$

1.4

Since the neural activities change continuously in time therefore the columns of $\boldsymbol{r}$ (which correspond to the population activity at different times) evolve smoothly in the neural space. PCA provides a tool for visualizing the evolution of the neural traces in a low dimensional space. Run PCA on the complete duration of the simulation (both evoked and spontaneous activity) and project the neural traces onto the first 3 PCs. Visualize the neural trajectory in the 3-D PC space and color the spontaneous and the evoked activity differently. Describe your observations and conclusions.

2 [Optional] High Dimensional Data

This problem will give you experience with the (sometimes counter-intuitive) properties of high-dimensional spaces.

2.1

Plot the average angle ϕ between two randomly chosen N-dimensional vectors x and y as a function of N between 1 and 50. Assume that the elements of the vectors are chosen i.i.d. from a standard Gaussian distribution. Determine the angle using $\phi = \min(\phi', \pi - \phi')$, where $\cos\phi' = \frac{x \cdot y}{\|x\|\|y\|}$.

2.2

Again as a function of N from 1 to 50, plot the probability that a randomly chosen point in the N-dimensional unit cube (a vector x whose elements $x_i, i = 1 \ldots N$ are chosen i.i.d. from a uniform distribution between -1 and 1) lies within the N-dimensional unit sphere. A vector x lies within the unit sphere if $\sum_{i=1}^{N} x_i^2 < 1$.

References

[1] Larry F Abbott, Kanaka Rajan, and Haim Sompolinsky. Interactions between intrinsic and stimulus-evoked activity in recurrent neural networks. *The dynamic brain: an exploration of neuronal variability and its functional significance*, pages 1–16, 2011.