$y = \underline{r}\,\frac{\psi}{c}$ ← recurrent weight strength; c ← External input strength

$$\tau \frac{dy}{dt} = -y + \alpha\,(Jy + g)_+^{n} \qquad J = \frac{W}{\psi}$$

$\alpha = \psi c^{n-1}$ $\qquad g = \frac{h}{c}$

$\psi \sim \|W\|$

$c = \|h\|$

recurrently: ~~$W \gg c$~~ → $\alpha \gg 1$

feedforward: ~~$W \ll c$~~ → $\alpha \ll 1$

$\frac{c}{\psi}$

$$\underline{r} = \frac{c}{\psi}\, y$$

When balanced?

$f(x) = 0$ if $x \le 0$

$\psi J \leftarrow W$

$\frac{cy}{\psi} \leftarrow r$

$$\underline{r} = f(W\underline{r} + cg)$$

$$\frac{cy}{\psi} = f\left(\psi J \frac{cy}{\psi} + cg\right)$$

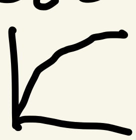
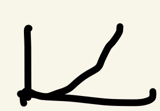
$$= f(c(Jy + g))$$

$$-g + \frac{1}{c} f^{-1}\left(\frac{cy}{\psi}\right) = (Jy)$$

$$y = -J^{-1}g + J^{-1}\frac{1}{c} f^{-1}\left(\frac{cy}{\psi}\right)$$

(1) if scale c & ψ together

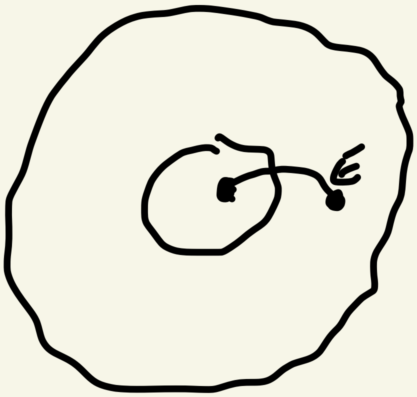
(2) if scale c & $\frac{1}{c} f^{-1}(c) \underset{c\to\infty}{\longrightarrow} 0$

$f^{-1}(c)$  $f(c)$ 

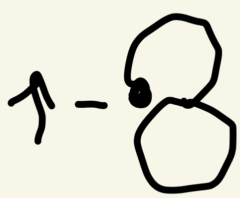
SSN ① Explaining all behaviours described previously

- ? sublinear summation
  ⇒ linear for weaker inputs

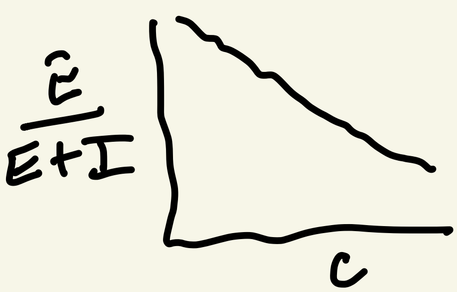
Sanzeni, ..., Brunel, Histed



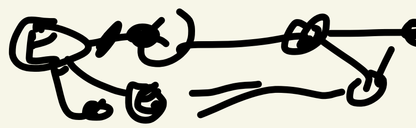
- Connectivity decreases w/ distance
  surround suppression/facilitation



- shared variability of neural response decreases w/ stimulus



- chaotic
  stimulus → suppresses chaos
  Litwin-Kumar
- Ashok & Doiron



clusters

Ahmadian 2017

Attractors

fixed input → $t \to \infty$: fixed point, Oscillations, Chaos

multiple fixed point

Hopfield $\underline{u}^i$ $i, \ldots p$ uncorrelated (0, 1, 0 0 1 ...)

"batch" $\boxed{W = \sum_i \underline{u}^i \underline{u}^{iT}}$

$$Wr = \sum_i \underline{u}^i \underbrace{u_i^T \underline{u}_j}_{\delta_{ij}} \to \underline{u}_j$$

N = # neurons

memories $\propto N$

"online" Stefano → # memories $\log N$

assuming finite # levels synaptic strength

forget e^{-kt}

Lyapunov function $f(\underline{r})$

"Energy" function

$\frac{df(\underline{r})}{dt} < 0$ & has a minimum value

$$\tau \frac{dr}{dt} = -r + W\underline{r} + \underline{h}$$

$$L = \frac{1}{2} \underline{r}^T W \underline{r} + \underline{r}^T \underline{h} - \underline{r}\underline{r}$$

$\nabla_{\underline{r}} L$

$$\frac{dL}{d\underline{r}} = \frac{1}{2} W\underline{r} + \frac{1}{2}\underline{r}^T W + \underline{h} - \underline{r}$$

if $W_{ij} = W_{ji}$

$(W\underline{r})_i = \sum_j W_{ij} r_j$

$(\underline{r}^T W)_i = \sum_j r_j W_{ji} = \sum W_{ij} r_j$

$$= W\underline{r} + \underline{h} - \underline{r}$$

$$\frac{dL}{d\underline{r}} = \frac{dr}{dt} \qquad \frac{dL}{dt} = \frac{dL}{d\underline{r}} \frac{d\underline{r}}{dt} = \left(\frac{dr}{dt}\right)^2$$

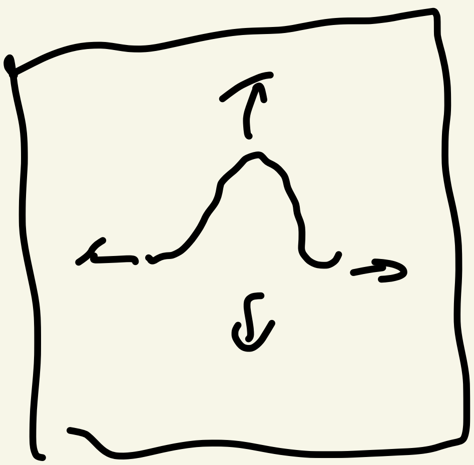
Attractors

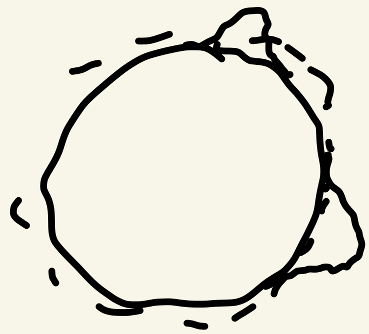
"Bump" attractor

"Ring" attractor

Continuous attractor

W





$$\frac{dr}{dt} = \underbrace{-r + Wr}_{\lambda_W = 1} + h$$

$$\frac{dr_i}{dt} = h_i$$

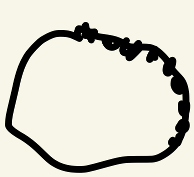
- working memory
- integration ←
  - path integration
    - place — grid
    - heading direction
    - oculomotor integrator

Problem: fragile

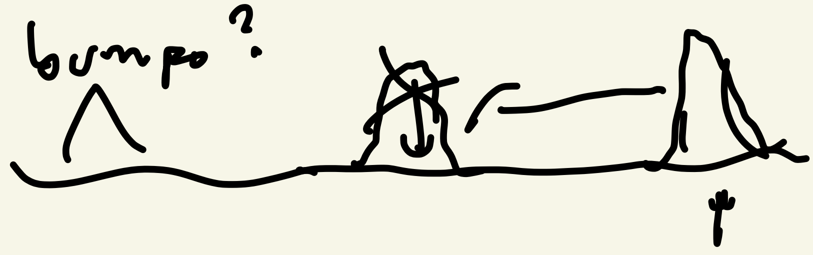
$\lambda = 0$

$\lambda > 0 \Rightarrow$ drift, explode

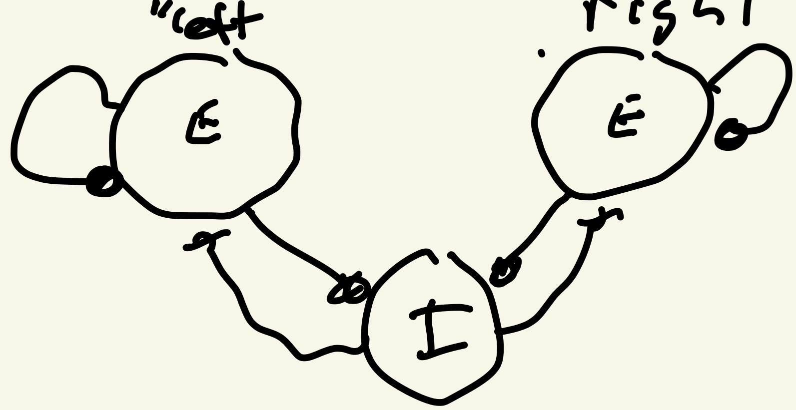
$\lambda < 0 \Rightarrow$ drift, diminish



two bumps?



~decision making



Models of activity-dependent development
~ "learning"

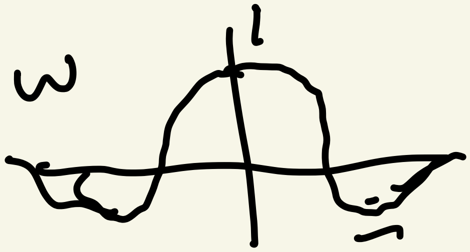
Von der Malsburg '73

(1) Hebbian rule "synapses that fire together wire together"

Correlation-based: causal ↑ corr between pre & post

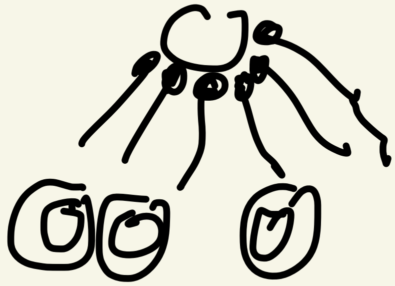
Competitive: if one corr pattern ↑↑ others must go ↓↓
⇒ post cell selective

(2) Cortical Activity spatially clustered
⇒ nearby cells develop correlated RFs



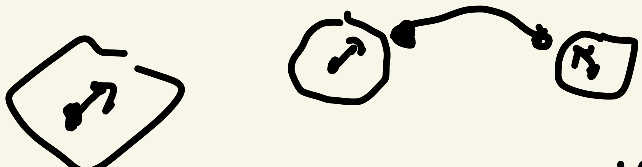
Two approaches:

(1) "High D"



(2) "Feature-based"

Assume cells represent a feature



Fred Wolf

pattern formation

- pattern forming instability selects period
- near instability
  "universality classes" based on symmetries

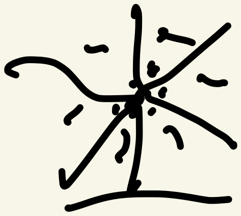
$z = re^{i2\theta}$

θ = pref or

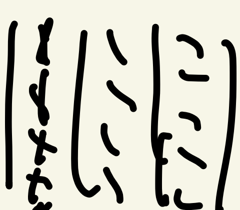
r = strength of ori selectivity

- translation
- rotation (spatial)
- rotation (orientations)

combined



when does solution arise w/ stable pinwheels?



Long-range suppression interaction

> 1 cycle of ORI period

λ

↓

density of pinwheels per λ^2

$\approx \pi$

20,000 pinwheels ~ 10

100 maps $\pi \pm 2\%$

Two primates

Carnivore