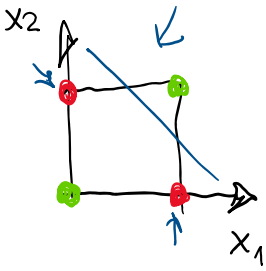
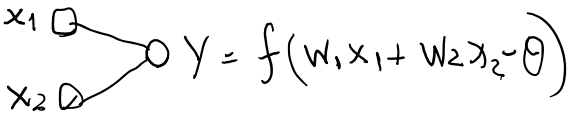
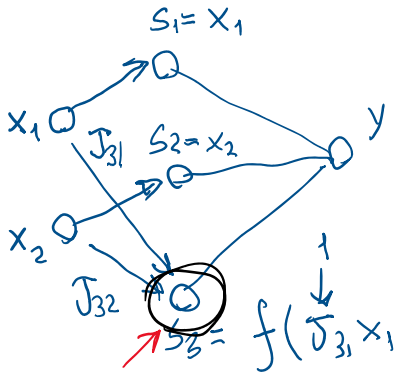
Multilayer networks (lecture)

Thursday, April 23, 2020 13:20





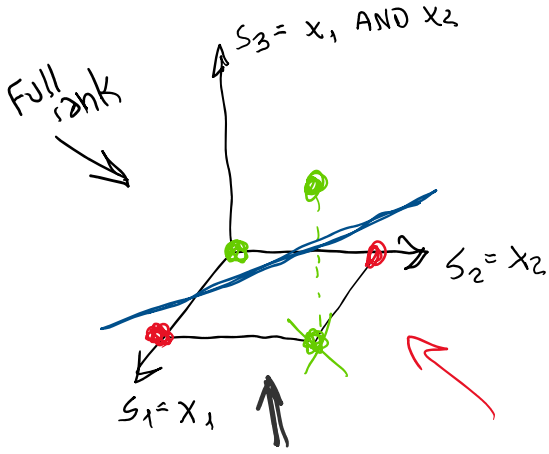
x_1	x_2	x_1 XOR x_2
0	1	1
1	0	1
1	1	0
0	0	0



$$s_3 = f(J_{31}x_1 + J_{32}x_2 - \theta)$$

$$f(x_1 + x_2 - 1.5) = x_1 \text{ AND } x_2$$

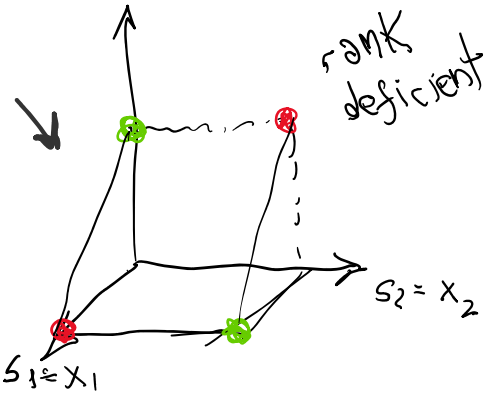
$$f \rightarrow H$$



$$s_3 = J_{31}x_1 + J_{32}x_2 - \theta$$

$$s_4 = J_{41}x_1 \dots$$

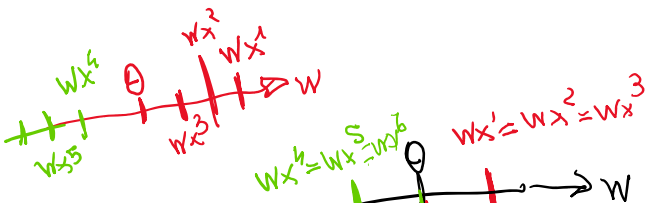
$$s_5 =$$



$$X = \begin{pmatrix} \vdots & \vdots & \dots \\ \underline{x}^1 & \underline{x}^2 & \\ \vdots & \vdots & \end{pmatrix} \quad N \times p$$

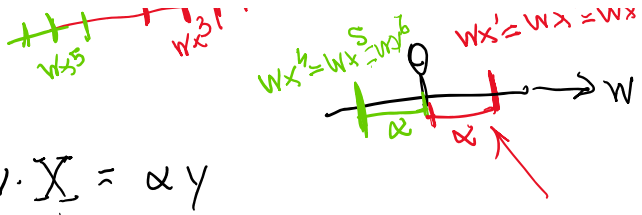
p

$$\Rightarrow y^\mu = f(\underline{w}\,\underline{x}^\mu - \theta) \quad \forall \mu$$



$$y^\mu = f(\underline{w}\,\underline{x}^\mu - \theta) \quad \forall \mu$$

$$y^\mu = (\underline{w}\,\underline{x}^\mu - \theta)\cdot \alpha, \quad \alpha = 1$$



$$w \cdot \underline{X} = \alpha y$$

$$(w_1 \dots w_N)\begin{pmatrix} & \\ & \underline{X} \\ & \end{pmatrix} = (y^1 \dots y^N)$$

$1 \times N$ $\quad$ $N \times p$

$p \leq N$

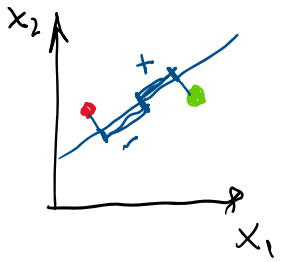
$$w\,\underline{X}\,\underline{X}^{-1} = \alpha y \underline{X}^{-1}$$

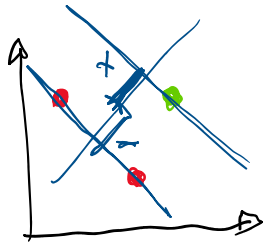
$$I$$

$$\boxed{w = \alpha y \underline{X}^{-1}}$$

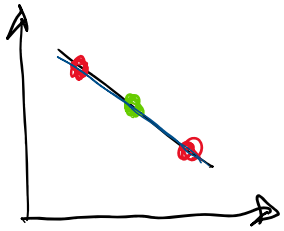
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ Full rank

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

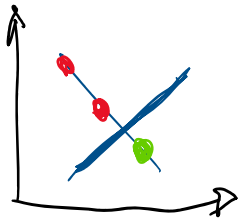




A



B



Max D

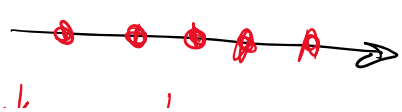
$$N\begin{pmatrix} | & | \\ \underline{x}^1 & \underline{x}^2 \\ | & | \end{pmatrix}$$

p

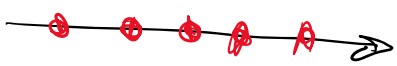
$N \times p$

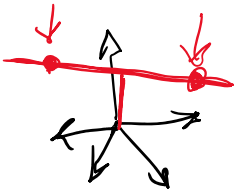
$rank(X) \leq \min(N, p)$

1) $N < p$

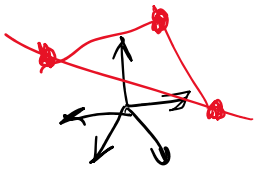


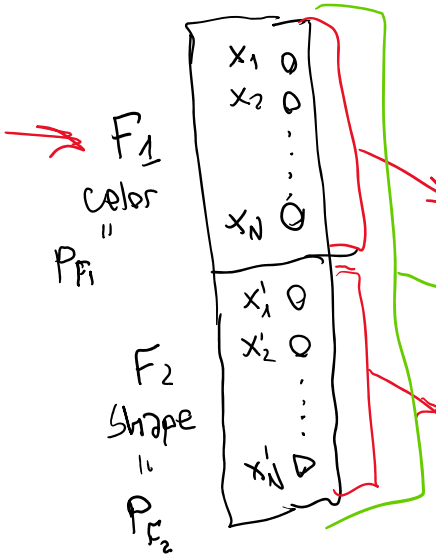
$D \leq N$

1) $N < p$  $D \leq N$

2) $p < N$  $p = 2 \Rightarrow D = 1 + 1$ (↑ offset)

$\max D \leq p$

 $p = 3 \Rightarrow D = 2 + 1$



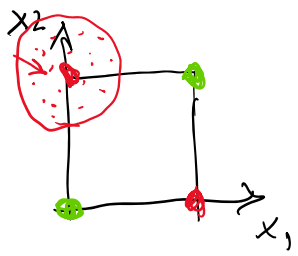
$\max D = p_{F_1} - 1$

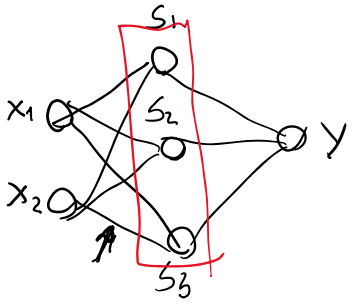
$p_{f_1} * p_{F_2} - 2 + 1 =$

$\max D = p_{F_2} - 1$

$p_{F_1} + p_{f_2} - 1$

$p = p_{F_1} \cdot p_{F_2} \gg p_{F_1} + p_{f_2} - 1$



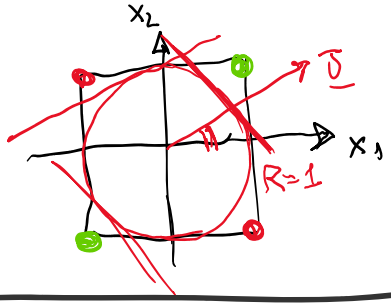


$S_k = f\left(\sum_{1,j}^{N} J_{kj} x'_j - \theta_k\right)$

↑ Random

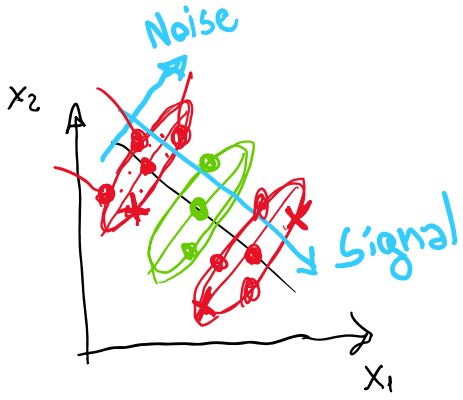
$\underline{J} \cdot \underline{x}$

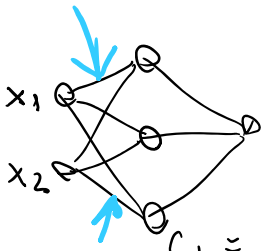
$J_{11}^2 + J_{12}^2 = R^2$



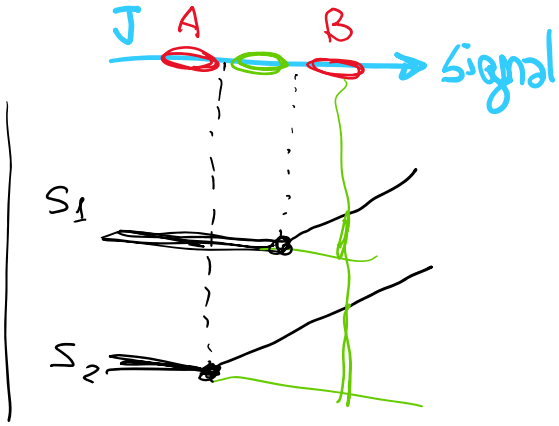
Noise

1.0

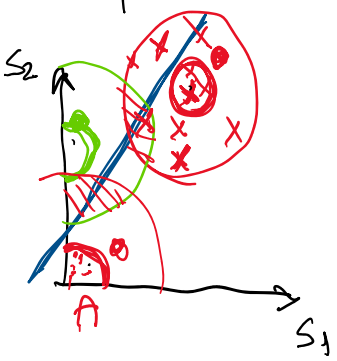


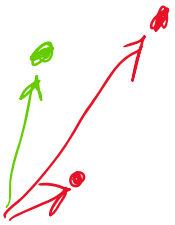


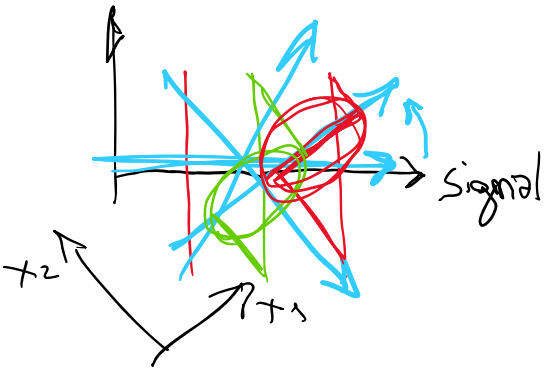
$$s_k = f(\underline{J} \cdot \underline{x} - \theta)$$



$$s_1 = f(J_{signal} \cdot \underline{x} - \theta)$$





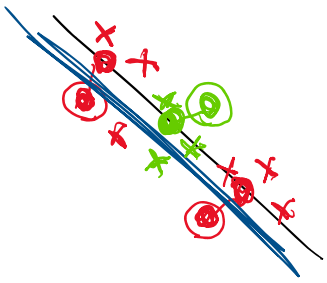


$$X = \left(\begin{array}{|c|c|c|c|}\hline \\ \hline \end{array}\right)$$

$$S = \left(\begin{array}{|c|c|c|c|}\hline \\ \hline \end{array}\right)$$

$\text{rank } S \geq \text{rank } X$

$$N \left(\begin{array}{|c|c|c|} \hline x_1 & x_2 & \cdots \\ \hline \end{array} \right) p$$



Rigotti et al 2013