Hopfield and forgetting

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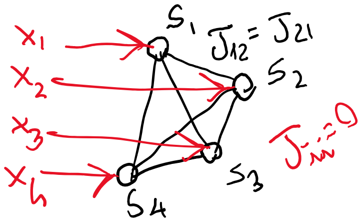


$$\boxed{s_i(t+1) = f\left(\sum_j^N J_{ij} s_j(t) + x_i(t)\right)}$$

f = sign

$$\underline{x} = \underline{\xi}^1, \underline{\xi}^2, \dots \underline{\xi}^P$$

$$J_{ij}(0) \quad J_{ij}(1) \quad \dots \quad J_{ij}(P)$$

$$\underline{s}(t \to \infty)$$

$$J_{ij} \qquad \underline{s}(t) = \underline{\tilde{\xi}}^\mu$$

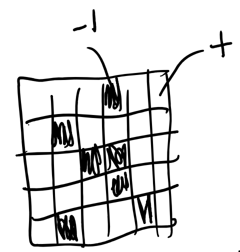
$$\underline{s}(t+1) = \underline{\xi}^\mu$$

$$\underline{\xi}^2 \to \underline{s} \to \underline{\xi}^2$$

$$\text{Cue } \underline{\tilde{\xi}}^2 \to \underline{s} \to \underline{\xi}^2$$

$$\underline{\xi}^1 \dots \underline{\xi}^P$$

$$\xi_i^\mu = \pm 1$$



$$\underline{\xi}^1 = (+ + +, \dots +)$$

$$\underline{\xi}^2 = (- - -, \dots, -)$$

$$s_i(t+1) = f\left(\sum_j^N J_{ij} s_j(t)\right)$$

$$J_{ij} = +1$$

$$\underline{\tilde{\xi}}^1 = (+, -, +, \dots, +)$$

$$= f\left(\sum_j^N s_j(t)\right)$$

$$E(\{s_1 \dots s_N\}) = -\sum_{i,j} s_i J_{ij} s_j = E(t)$$

$$\Rightarrow s_1(t+1) = \text{sign}\Big(\underbrace{\sum_j^N J_{ij} s_j(t)}_{h_i(t)}\Big) \Rightarrow E(t+1) \leq E(t)$$

$$\boxed{J_{ij} = J_{ji}} \qquad \boxed{J_{ii} = 0}$$

$$\Delta E = E(t+1) - E(t) = -\sum_{i,j} s_i(t+1) J_{ij} s_j(t+1) + \sum_{i,j} s_i(t) J_{ij} s_j(t)$$

$$= -\sum_{j=2}^N s_1(t+1) J_{1j} s_j(t+1) - \sum_{i=2}^N s_i(t+1) J_{i1} s_1(t+1) +$$

$$+ \qquad (t) \qquad (t) \qquad (t) \qquad (t)$$

$J_{ij} = J_{ji}$

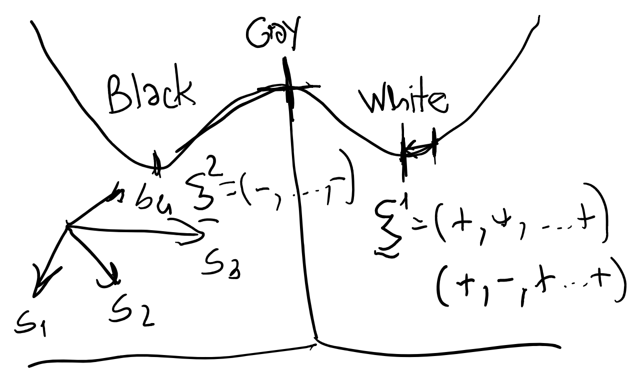
$$\Delta E = -2\left(s_1(t+1) - s_1(t)\right)\underbrace{\left(\sum_{j=2}^{N} J_{1j} s_j(t)\right)}_{h_1(t)}$$

$$\boxed{\Delta E = -2\,\Delta s_1 \cdot h_1(t)}$$

1. $\Delta s_1 = 0 \Rightarrow \Delta E = 0$
2. $\Delta s_1 \neq 0 \Rightarrow \underline{s_1(t+1)} = \text{sign}(h_1(t)) = \underline{-s_1(t)}$

$$\Delta s_1 = \text{sign}(h_1) - \left(-\text{sign}(h_1)\right) = 2\,\text{sign}(h_1)$$

$$\Delta E = -2\,\text{sign}(h_1) \cdot h_1 = -2|h_1| < 0$$



$\underline{\xi}^1 = \{+,-,+,\ldots +\}$

$\underline{\xi}^2 = \{-,+,-,\ldots -\}$

$$s_i' = \underline{\overset{\pm 1}{\xi_i^1} \cdot \overset{\pm 1}{s_i}}$$

$(+,+,+,\ldots +)$

$(- - - \quad -)$

$s_i = \xi_i^1$

$s_i' = \xi_i^1 \xi_i^1 = (\xi_i^1)^2 = +1$

$$s_i'(t+1) = \text{sign}\left(\sum_j^N \overset{+1}{\overset{\shortparallel}{J'_{ij}}} s_j'(t)\right)$$

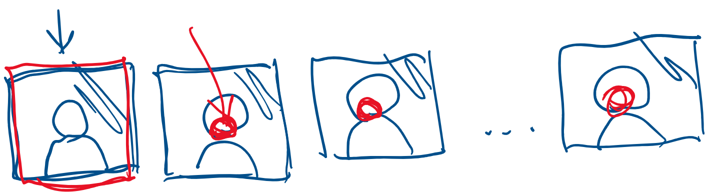
$$\cancel{\underset{+1}{\xi_i^1 \cdot \xi_i^1}}\, s_i(t+1) = \text{sign}\left(\sum_{j}^{N} \xi_j^1 s_j(t)\right) \cdot \overset{\pm 1}{\xi_i^1}$$

$$s_i(t+1) = \mathrm{sign}\left(\sum_j^N \xi_i^1 \xi_j^1 s_j(t)\right)$$

$$J_{ij} = \xi_i^1 \xi_j^1 \qquad \xi_i^2 = -\xi_i^1$$

$$\rightarrow J_{ij} = \sum_\mu^P \xi_i^\mu \xi_j^\mu$$

ξ_i^μ Random uncorrelated $\rightarrow \langle \xi_i^\mu \xi_j^\nu \rangle = 0 \quad \forall \mu \neq \nu,\ i \neq j$



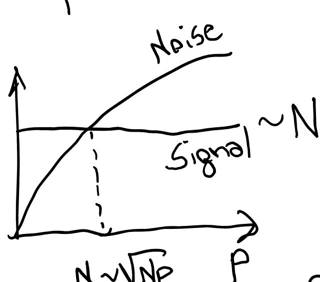
$$h_i = \sum_{j\neq i} J_{ij} \xi_j^1 = \sum_{j\neq i} \sum_\mu^P \xi_i^\mu \xi_j^\mu \xi_j^1 =$$

Signal

$$= \sum_j \xi_i^1 \underbrace{\xi_j^1 \xi_j^1}_{+1} + \sum_{j\neq i}^{N-1} \sum_{\mu>1}^{P-1} \overbrace{\xi_i^\mu \xi_j^\mu \xi_j^1}^{\pm 1}$$

$\sim Np$ Noise

$$N\xi_i^1 + \eta \rightarrow (0, \sqrt{Np})$$



$p \sim N$

$$\boxed{p \sim N}$$

Amit Gutfreund Sompolinsky

$p = 0.14N$

$p > 0.14N \Rightarrow$ Blackout catastrophe

catastrophe

$$J_{ij} = \sum_{\mu}^{p} \xi_i^{\mu} \xi_j^{\mu} - \begin{cases} +p \\ \\ -p \end{cases} \qquad p \sim N$$

$$0.16N$$

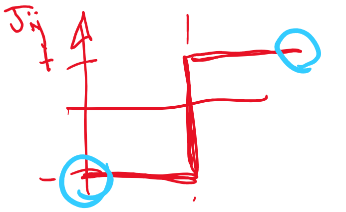
$$J_{ij} = \mathrm{sign}\left(\sum_{\mu}^{p} \xi_i^{\mu} \xi_j^{\mu} \right) = \pm 1 \quad \Rightarrow p \sim N \qquad p = 0.08N$$

$$\boxed{J_{ij} = \pm 1}$$

M

$$\mathrm{Signal}^1 = \sum_i^N \overbrace{\xi_i^1 h_i^1} = \sum_{i,j} \xi_i^1 J_{ij} \xi_j^1 =$$

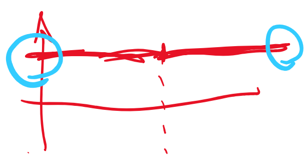
$$= \sum_{i,j} \underbrace{\left(\xi_i^1 \xi_j^1 \right)}_{\substack{\shortparallel \\ \Delta J_{ij}^1 \\ \shortparallel \\ \pm 1}} \cdot \underbrace{J_{ij}}_{\substack{\shortparallel \\ \pm 1}}$$



$$\Delta J_{ij}^1 = +1$$

$$qM \cdot (1-q)\,(1-q) = qM(1-q)^{p-1}$$

$$\boxed{\xi^1} \qquad \xi^2 \qquad \xi^3$$



$$J_{ij} = \underline{\xi_i^1 \xi_j^1}$$

$$\Delta J_{ij}^1 = +1$$

$$qM(1-q)^p = qM e^{\log(1-q)^p} = qM e^{p \log(1-q)} =$$

$$\boxed{\simeq qM e^{-pq}}$$

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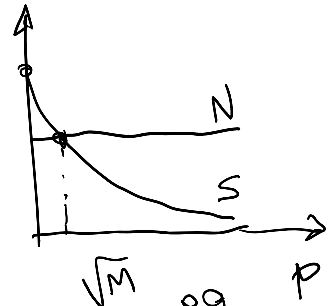
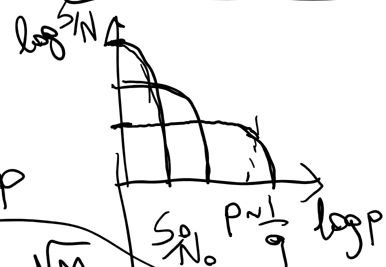
Signal $\sim q M e^{-pq}$

Noise $\sim \sqrt{M}$



$\sqrt{M} \sim q \cancel{M} e^{-pq}$

$\boxed{p \sim N} \rightarrow \boxed{p \sim \frac{1}{q} \log q\sqrt{M}}$



$\frac{S}{N} \sim q\sqrt{M} e^{-pq}$

p

Slow : $q \ll 1$	$q \sim \frac{1}{\sqrt{M}}$	$p \sim \sqrt{M}$	$\sim O(1)$
Fast : $q \sim 1$		$p \sim \log M$	$\sqrt{M}$