Assignment 4

1. *Linearizing a nonlinear feedforward network*

   Consider a one-layer feedforward network with input $\mathbf{x}$, connection matrix W, and output $\mathbf{r} = f(W\mathbf{x})$. By taking partial derivatives of the elements of $\mathbf{r}$ with respect to those of $\mathbf{x}$, show that, if $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}_0$ is small, then $\Delta\mathbf{r} = \mathbf{r} - \mathbf{r}_0 = \mathbf{r} - f(W\mathbf{x}_0)$ can be approximated as:

   $$\Delta\mathbf{r} \approx DW\Delta\mathbf{x}, \tag{1}$$

   where D is a diagonal matrix. What are the elements of D?

2. *A random nonlinear firing rate network*

   Suppose we have a nonlinear firing-rate network that evolves according to:

   $$\dot{\mathbf{x}}(t) = -\mathbf{x}(t) + Wf(\mathbf{x}(t)), \tag{2}$$

   where W is an $N \times N$ matrix of connections with $N = 500$ and each element of W drawn from a normal distribution with standard deviation $g/\sqrt{N}$ (in MATLAB, `W = g*randn(N)/sqrt(N)`). Further assume that f is the tanh function, applied to each element of the vector $\mathbf{x}$.

   1) As discussed in class, the network dynamics linearized around the fixed point $\mathbf{x} = \mathbf{0}$ are:

   $$\dot{\mathbf{x}}(t) = (-I + W)\mathbf{x}(t). \tag{3}$$

   Compute the eigenvalues of the matrix $-I + W$ numerically and make a scatter plot of their real and imaginary parts in the complex plane, for $g = 0.9$ and $g = 1.1$.

   2) Simulate the linearized network for 100 time units and plot the activity of 5 example neurons for $g = 0.9$, $g = 1.1$. Choose the initial condition $\mathbf{x}(0)$ to be a vector of elements each drawn from a standard normal distribution.

   3) Do the same as part 2 but for 500 time units of the nonlinear network's dynamics. Also plot the behavior for $g = 2$.

3. *Optional: Analysis of the FitzHugh-Nagumo model*

   The FitzHugh-Nagumo model is a 2-dimensional neuron model that evolves according to:

   $$\dot{v} = v - v^3/3 - w + I \tag{4}$$
   $$\tau\dot{w} = v + a - bw. \tag{5}$$

The v variable represents membrane voltage, w a recovery variable that provides negative feedback to v, and I the magnitude of externally applied current. Assume $\tau = 12.5$, $a = 0.7$, $b = 0.8$.

1) Set $\dot{w} = 0$ and $\dot{v} = 0$ to obtain a cubic polynomial in v whose solution determines the location of the fixed point. Write a function that finds the (v, w) pair corresponding to this fixed point.

2) Simulate the system for different values of external current I and plot the time evolution of the voltage variable v. Set the initial condition to be close to the fixed point found in part 1, but with a depolarized voltage. Find values of I that correspond to quiescence and to repetitive spiking.

3) Write down the Jacobian for the two-dimensional linearized system.

4) Using the function you wrote in part 1, find the eigenvalues of the linearized system evaluated at the fixed point and use this to determine the value of I for which the fixed point becomes unstable. Check that this matches the behavior you observe in your simulations.