I just wanted to briefly correct the point I got hung on re Lyapunov functions in today’s lecture. You’ll recall we had the equation

\[ \frac{dv}{dt} = -v + Wf(v) + h \]  

(1)

with \( W \) symmetric. We asserted the Lyapunov function

\[ E = -\sum_i \int_0^{r_i} f^{-1}(x)dx + \frac{1}{2}r^TWr + h^Tr \]  

(2)

In class, I got confused by the fact that there were vectors in the last two terms for \( E \) but only individual elements in the first term. The last two terms are scalars, unchanged by any coordinate transformation to orthonormal basis vectors (i.e., by an orthogonal transformation); but the first, which depends on individual elements of \( r \), depends on the coordinate system. But this is easily fixed; if the orthonormal basis vectors of the current coordinate system are \( e_i \), so that \( r = \sum_i r_ie_i \), then we can replace the first term with \( \sum_i \int_0^{r_i} e_i f^{-1}(x)dx \), which is invariant under orthogonal coordinate transformations. And then we can choose to do the calculation in the current coordinate system, in which case the equation for \( E \) becomes Eq. 2.